Guaranteed Cost Controller Design of Networked Control Systems with State Delay

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Abstract This paper is concerned with the state-feedback guaranteed cost controller design for a class of networked control systems (NCSs) with state-delay. A new model of the NCSs is provided under consideration of the network-induced delay. A sufficient condition for the existence of a guaranteed cost controller for NCSs is presented by a set of linear matrix inequalities(LMIs). A method, which can transform non-convex to the convex, is applied. Accordingly, a numerical algorithm is proposed to obtain the lower bound. Theoretical analysis through an example shows the effectiveness of the method.

Key words Networked control systems, guaranteed cost control, linear matrix inequalities

1 Introduction

The networked control systems(NCSs) have recently been studied by more and more researchers since their low cost, reduced weight and power requirements, simple installation and maintenance and high reliability[1,2]. In an NCS, one of the important issues to treat is the effect of the network-induced delay on the system performance. For the NCSs with different scheduling protocols, the network-induced delay may be constant, time-varying, or even random variable[3].

The guaranteed cost control of uncertain systems was first put forward by [4] and studied by a lot of researchers, which is to design a controller to robustly stabilize the uncertain system and guarantee an adequate level of performance. The guaranteed cost control approach has recently been extended to the uncertain time-delay systems, for the state feedback cases, see [5∼7]; for the output feedback case, see [8].

In this paper, the author′s intention is to design a guaranteed cost controller based on the network delay for a class of uncertain time-delay. Sufficient condition for the existence of a guaranteed cost state-feedback controller is established in terms of matrix inequalities. At the same time, the maximum allowable value τ_{max} of the network-induced and guaranteed cost bound are obtained and the guaranteed cost control strategy is proven by a numerical example.

The remainder of this paper is organized as follows: Section 2 describes the problem formulation. In Section 3, details of the modeling of networked control systems are discussed. Section 4 obtains the guaranteed cost controller design and illuminates the method which switches a non-convex problem to a convex. Section 5 introduces a numerical example. Section 6 presents conclusions.

2 Problem formulation

Consider a class of linear uncertain system with time-delay in the state described by the following equations

$$\left\{\begin{array}{l} x'(t) = [A+\Delta A]x(t) + [A_d + \Delta A_d]x(t-d) + \\ \qquad [B+\Delta B]u(t) \\ x(t) = \varphi(t), t \in [-d, 0] \end{array}\right. \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $A, A_d \in R^{n\times n}$ and $B \in R^{n\times m}$ are known constant real matrices, $\Delta A, \Delta A_d$ and ΔB are matrix-valued functions of appropriate dimension parameter uncertainties in the system model. $\varphi(t)$ is a given continuous vector-valued initial function and subsection differential on $[-d, 0]$. The parameter uncertainties considered are assumed to be norm bounded and satisfy

$$[\Delta A \;\; \Delta A_d \;\; \Delta B] = DF(t)[E \;\; E_d \;\; E_b] \tag{2}$$

where D, E, E_d, E_b are known constant real matrices of appropriate dimensions that represent the structure of uncertainties, and $F(t) \in R^{i\times j}$ is an uncertain matrix function with Lebesgue measurable elements and satisfies

$$F^{\mathrm{T}}(t)F(t) \le I \tag{3}$$

in which I denotes the identity matrix of appropriate dimension. Associated with system (1) is the cost function

$$J = \int_0^{\infty} [x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)]\mathrm{d}t \tag{4}$$

where Q and R are given positive-definite symmetric matrices.

Definition[9]. For the uncertain system (1), if there exist a control law $u^*(t)$ and a positive scalar J^* such that for all admissible uncertainties the closed-loop system is stable and the closed-loop value of the cost function (4) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law for the uncertain system (1).

The objective of this paper is to develop a procedure for designing a memoryless state feedback guaranteed cost control law

$$u(t) = Kx(t) \tag{5}$$

for the linear uncertain time-delay system (1).

3 Modeling of networked control systems

In an NCS, suppose that the sensor is clock-driven, the controller and actuator are event-driven and the data is transmitted with a single-packet. Then the real input $u(t)$ realized through zero-order hold in (1) is a piecewise constant function. Furthermore, if we consider the effect of the network-induced delay and network packet dropout on the NCSs, then the real control system (1) with (5) can be rewritten as

$$\left\{\begin{array}{l} x'(t) = [A+\Delta A]x(t) + [A_d + \Delta A_d]x(t-d) + \\ \qquad [B+\Delta B]u(t), t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}) \\ u(t^+) = Kx(t-\tau_k), t \in \{i_k h + \tau_k, \cdots\} \end{array}\right. \tag{6}$$

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where h is the sampling period, $i_k(k=1,2,\cdots)$ are some integers and $\{i_1, i_2, \cdots\} \subseteq \{1,2,\cdots\}, \tau_k$ is the time delay, which denotes the time from the instant $i_k h$ when the sensor nodes sample sensor data from a plant to the instant when actuators transfer data to the plant. Obviously, $\bigcup_{k=1}^{\infty}[i_k h+\tau_k, i_{k+1}h+\tau_{k+1}) = [0,\infty)$. According to (2), system (6) can be rewritten as the following equivalent form

$$x'(t) = [A+DF(t)E]x(t) + [A_d + DF(t)E_d]x(t-d) + [B+DF(t)E_b]Kx(i_k h), t \in [i_k h+\tau_k, i_{k+1}h+\tau_{k+1}) \tag{7}$$

and (4) can be rewritten as

$$J = \int_0^{\infty} x^{\mathrm{T}}(t)Qx(t)\mathrm{d}t + \sum_{k=1}^{\infty}\int_{i_k h+\tau_k}^{i_{k+1}h+\tau_{k+1}} x^{\mathrm{T}}(i_k h)K^{\mathrm{T}}RKx(i_k h)\mathrm{d}t \tag{8}$$

4 Main results

The following Lemmas will be used.

Lemma 1. For any vectors a, b and positive-definite matrix X, there exists

$$\pm 2a^{\mathrm{T}}b \le a^{\mathrm{T}}X^{-1}a + b^{\mathrm{T}}Xb$$

Lemma 2. Given appropriate dimension matrices D, E and symmetric Y, the matrix inequality

$$Y + DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} < 0$$

holds for all F satisfying $F^{\mathrm{T}}F \le I$ if and only if there exists a constant $\varepsilon > 0$ such that

$$Y + \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0$$

First we present a sufficient condition for the existence of memoryless state feedback guaranteed cost control laws for an uncertain time-delay system.

Theorem 1. $u(t) = Kx(t), (K = YX^{-\mathrm{T}})$ is a guaranteed cost controller if there exist symmetric positive-definite matrices $\vec{P} > 0, \vec{S} > 0, \vec{T} > 0$, and appropriate dimension matrices $X, Y, \vec{N}_i (i=1,2,3,4)$, and a scalar $\varepsilon > 0$ for any given scalars $\tau, \rho_i (i=2,3,4)$, and matrices $Q > 0, R > 0$ such that (9) holds. At the same time the guaranteed cost J^* satisfies (11).

$$(i_{k+1} - i_k)h + \tau_{k+1} \le \tau, k = 1,2,\cdots \tag{10}$$

$$J \le \varphi^{\mathrm{T}}(0)X^{-1}\vec{P}X^{-\mathrm{T}}\varphi(0) + \int_{-d}^{0}\varphi^{\mathrm{T}}(\alpha)X^{-1}\vec{S}X^{-\mathrm{T}}\varphi(\alpha)\mathrm{d}\alpha + \int_0^{\tau}\int_{-\beta}^{0}\varphi'^{\mathrm{T}}(\alpha)X^{-1}\vec{T}X^{-\mathrm{T}}\varphi'(\alpha)\mathrm{d}\alpha\mathrm{d}\beta = J^* \tag{11}$$

where

$$\begin{cases}
\Omega_{11} = \vec{S} + \vec{N}_1 + \vec{N}_1^{\mathrm{T}} - AX^{\mathrm{T}} - XA^{\mathrm{T}} + \varepsilon DD^{\mathrm{T}} \\
\Omega_{21} = \vec{N}_2 - XA_d^{\mathrm{T}} - \rho_2 AX^{\mathrm{T}} + \varepsilon\rho_2 DD^{\mathrm{T}} \\
\Omega_{22} = -\vec{S} - \rho_2 XA_d^{\mathrm{T}} - \rho_2 A_d X^{\mathrm{T}} + \varepsilon\rho_2^2 DD^{\mathrm{T}} \\
\Omega_{31} = -\vec{N}_1^{\mathrm{T}} + \vec{N}_3 - Y^{\mathrm{T}}B^{\mathrm{T}} - \rho_3 AX^{\mathrm{T}} + \varepsilon\rho_3 DD^{\mathrm{T}} \\
\Omega_{32} = -\vec{N}_2^{\mathrm{T}} - \rho_2 Y^{\mathrm{T}}B^{\mathrm{T}} - \rho_3 A_d X^{\mathrm{T}} + \varepsilon\rho_2\rho_3 DD^{\mathrm{T}} \\
\Omega_{33} = -\vec{N}_3 - \vec{N}_3^{\mathrm{T}} - \rho_3 BY - \rho_3 Y^{\mathrm{T}}B^{\mathrm{T}} + \varepsilon\rho_3^2 DD^{\mathrm{T}} \\
\Omega_{41} = \vec{N}_4 + X - \rho_4 AX^{\mathrm{T}} + \vec{P} + \varepsilon\rho_4 DD^{\mathrm{T}} \\
\Omega_{42} = \rho_2 X - \rho_4 A_d X^{\mathrm{T}} + \varepsilon\rho_2\rho_4 DD^{\mathrm{T}} \\
\Omega_{43} = -\vec{N}_4 + \rho_3 X - \rho_4 BY + \varepsilon\rho_3\rho_4 DD^{\mathrm{T}} \\
\Omega_{44} = \tau\vec{T} + \rho_4 X + \rho_4 X^{\mathrm{T}} + \varepsilon\rho_4^2 DD^{\mathrm{T}}
\end{cases}$$

Proof. Construct a Lyapunov functional as

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{-d}^{t}x^{\mathrm{T}}(\alpha)Sx(\alpha)\mathrm{d}\alpha + \int_0^{\tau}\int_{-\beta}^{t}x'^{\mathrm{T}}(\alpha)Tx'(\alpha)\mathrm{d}\alpha\mathrm{d}\beta \tag{12}$$

where $P > 0, S > 0, T > 0$.

Since $x(t) - x(i_k h) - \int_{i_k h}^{t}x'(\alpha)\mathrm{d}\alpha = 0$ and by (7), one can see that any arbitrary matrices $N_i, M_i (i=1,2,3,4)$ of appropriate dimensions satisfy

$$2[x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-d)N_2 + x^{\mathrm{T}}(i_k h)N_3 + x'^{\mathrm{T}}(t)N_4] \times [x(t) - x(i_k h) - \int_{i_k h}^{t}x'(\alpha)\mathrm{d}\alpha] = 0 \tag{13}$$

$$2[x^{\mathrm{T}}(t)M_1 + x^{\mathrm{T}}(t-d)M_2 + x^{\mathrm{T}}(i_k h)M_3 + x'^{\mathrm{T}}(t)M_4] \times [-\bar{A}x(t) - \bar{A}_d x(i_k h) - \bar{B}Kx(i_k h) + x'(t)] = 0 \tag{14}$$

where $\bar{A} = A + DF(t)E, \bar{A}_d = A_d + DF(t)E_d, \bar{B} = B + DF(t)E_b$.

Taking the time derivative of $V(t)$ for $t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1})$, and using (13) and (14) yields

$$\begin{aligned}
V'(t) = {} & 2x^{\mathrm{T}}(t)Px'(t) + x^{\mathrm{T}}(t)Sx(t) + \tau x'^{\mathrm{T}}(t)Tx'(t) - \\
& x^{\mathrm{T}}(t-d)Sx(t-d) - \int_{-\tau}^{t}x'^{\mathrm{T}}(\alpha)Tx'(\alpha)\mathrm{d}\alpha + \\
& 2[x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-d)N_2 + x^{\mathrm{T}}(i_k h)N_3 + x'^{\mathrm{T}}(t)N_4] \times \\
& [x(t) - x(i_k h) - \int_{i_k h}^{t}x'(\alpha)\mathrm{d}\alpha] + \\
& 2[x^{\mathrm{T}}(t)M_1 + x^{\mathrm{T}}(t-d)M_2 + x^{\mathrm{T}}(i_k h)M_3 + x'^{\mathrm{T}}(t)M_4] \times \\
& [-\bar{A}x(t) - \bar{A}_d x(i_k h) - \bar{B}Kx(i_k h) + x'(t)]
\end{aligned} \tag{15}$$

$$\Omega = \begin{bmatrix}
\Omega_{11} & * & * & * & * & * & * & * \\
\Omega_{21} & \Omega_{22} & * & * & * & * & * & * \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * & * \\
\Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & * & * & * & * \\
\tau\vec{N}_1^{\mathrm{T}} & \tau\vec{N}_2^{\mathrm{T}} & \tau\vec{N}_3^{\mathrm{T}} & \tau\vec{N}_4^{\mathrm{T}} & -\tau\vec{T} & * & * & * \\
0 & 0 & Y & 0 & 0 & -R^{-1} & * & * \\
X^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & -Q^{-1} & * \\
EX^{\mathrm{T}} & E_d X^{\mathrm{T}} & E_b X^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix} < 0 \tag{9}$$

From (10) it can be seen that when $t \in [i_kh+\tau_k, i_{k+1}h+\tau_{k+1})$

$$-\int_{t-\tau}^{t} x'(\alpha)Tx'(\alpha)\mathrm{d}\alpha \le -\int_{i_kh}^{t} x'(\alpha)Tx'(\alpha)\mathrm{d}\alpha \qquad (16)$$

Using Lemma 1, we can show that

$$-2[x^{\mathrm{T}}(t)N_1 + x^{\mathrm{T}}(t-d)N_2 + x^{\mathrm{T}}(i_kh)N_3 + x'^{\mathrm{T}}(t)N_4]\times \int_{i_kh}^{t} x'(\alpha)\mathrm{d}\alpha \le \tau e^{\mathrm{T}}(t)NT^{-1}N^{\mathrm{T}}e(t) + \int_{i_kh}^{t} x'^{\mathrm{T}}(\alpha)Tx'(\alpha)\mathrm{d}\alpha \qquad (17)$$

where

$$e^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-d) \quad x^{\mathrm{T}}(i_kh) \quad x'^{\mathrm{T}}(t)]$$

$$N^{\mathrm{T}} = [N_1^{\mathrm{T}} \quad N_2^{\mathrm{T}} \quad N_3^{\mathrm{T}} \quad N_4^{\mathrm{T}}]$$

Combining (15), (16) and (17), we obtain

$$\begin{aligned} &V'(t) \le e^{\mathrm{T}}(t)\bar{\varOmega}e(t) - x^{\mathrm{T}}(t)Qx(t) - \\ &x^{\mathrm{T}}(i_kh)K^{\mathrm{T}}RKx(i_kh) \\ &t \in [i_kh+\tau_k, i_{k+1}h+\tau_{k+1}) \end{aligned} \qquad (18)$$

where

$$\bar{\varOmega} = \begin{bmatrix} \bar{\varOmega}_{11} & * & * & * & * \\ \bar{\varOmega}_{21} & \bar{\varOmega}_{22} & * & * & * \\ \bar{\varOmega}_{31} & \bar{\varOmega}_{32} & \bar{\varOmega}_{33} & * & * \\ \bar{\varOmega}_{41} & \bar{\varOmega}_{42} & \bar{\varOmega}_{43} & \bar{\varOmega}_{44} & * \\ \tau N_1^{\mathrm{T}} & \tau N_2^{\mathrm{T}} & \tau N_3^{\mathrm{T}} & \tau N_4^{\mathrm{T}} & -\tau T \end{bmatrix}$$

$$\begin{cases} \bar{\varOmega}_{11} = S + N_1 + N_1^{\mathrm{T}} - M_1\bar{A} - \bar{A}^{\mathrm{T}}M_1^{\mathrm{T}} + Q \\ \bar{\varOmega}_{21} = N_2 - M_2\bar{A} - \bar{A}_d{}^{\mathrm{T}}M_1^{\mathrm{T}} \\ \bar{\varOmega}_{22} = -S - M_2\bar{A}_d - \bar{A}_d{}^{\mathrm{T}}M_2^{\mathrm{T}} \\ \bar{\varOmega}_{31} = -N_1^{\mathrm{T}} + N_3 - K^{\mathrm{T}}\bar{B}^{\mathrm{T}}M_1^{\mathrm{T}} - M_3\bar{A} \\ \bar{\varOmega}_{32} = -N_2^{\mathrm{T}} - K^{\mathrm{T}}\bar{B}^{\mathrm{T}}M_2^{\mathrm{T}} - M_3\bar{A}_d \\ \bar{\varOmega}_{33} = -N_3 - N_3^{\mathrm{T}} - M_3\bar{B}K - K^{\mathrm{T}}\bar{B}^{\mathrm{T}}M_3^{\mathrm{T}} + K^{\mathrm{T}}RK \\ \bar{\varOmega}_{41} = N_4 + M_1^{\mathrm{T}} - M_4\bar{A} + P \\ \bar{\varOmega}_{42} = M_2^{\mathrm{T}} - M_4\bar{A}_d \\ \bar{\varOmega}_{43} = -N_4 + M_3^{\mathrm{T}} - M_4\bar{B}K \\ \bar{\varOmega}_{44} = \tau T + M_4 + M_4^{\mathrm{T}} \end{cases}$$

So if $\bar{\varOmega} < 0$, then (18) implies $V'(t) < 0$.

Using Lemma 2, we obtain

$$\begin{aligned} \bar{\varOmega} = \quad & \bar{Y} + M_d^{\mathrm{T}}FM_e + M_e^{\mathrm{T}}FM_d < 0 \\ \Leftrightarrow \quad & \bar{Y} + \varepsilon M_d^{\mathrm{T}}M_d + \varepsilon^{-1}M_e^{\mathrm{T}}M_e < 0 \end{aligned} \qquad (19)$$

where

$$\bar{Y} = \begin{bmatrix} \bar{Y}_{11} & * & * & * & * \\ \bar{Y}_{21} & \bar{Y}_{22} & * & * & * \\ \bar{Y}_{31} & \bar{Y}_{32} & \bar{Y}_{33} & * & * \\ \bar{Y}_{41} & \bar{Y}_{42} & \bar{Y}_{43} & \bar{Y}_{44} & * \\ \tau N_1^{\mathrm{T}} & \tau N_2^{\mathrm{T}} & \tau N_3^{\mathrm{T}} & \tau N_4^{\mathrm{T}} & -\tau T \end{bmatrix}$$

$$\begin{cases} \bar{Y}_{11} = S + N_1 + N_1^{\mathrm{T}} - M_1A - A^{\mathrm{T}}M_1^{\mathrm{T}} + Q \\ \bar{Y}_{21} = N_2 - M_2A - A_d^{\mathrm{T}}M_1^{\mathrm{T}} \\ \bar{Y}_{22} = -S - M_2A_d - A_d^{\mathrm{T}}M_2^{\mathrm{T}} \\ \bar{Y}_{31} = -N_1^{\mathrm{T}} + N_3 - K^{\mathrm{T}}B^{\mathrm{T}}M_1^{\mathrm{T}} - M_3A \\ \bar{Y}_{32} = -N_2^{\mathrm{T}} - K^{\mathrm{T}}B^{\mathrm{T}}M_2^{\mathrm{T}} - M_3A_d \\ \bar{Y}_{33} = -N_3 - N_3^{\mathrm{T}} - M_3BK - K^{\mathrm{T}}B^{\mathrm{T}}M_3^{\mathrm{T}} + K^{\mathrm{T}}RK \\ \bar{Y}_{41} = N_4 + M_1^{\mathrm{T}} - M_4A + P \\ \bar{Y}_{42} = M_2^{\mathrm{T}} - M_4A_d \\ \bar{Y}_{43} = -N_4 + M_3^{\mathrm{T}} - M_4BK \\ \bar{Y}_{44} = \tau T + M_4 + M_4^{\mathrm{T}} \\ M_d = -[(M_1D)^{\mathrm{T}} \quad (M_2D)^{\mathrm{T}} \quad (M_3D)^{\mathrm{T}} \quad (M_4D)^{\mathrm{T}} \quad 0] \\ M_e = [E \quad E_d \quad E_bK \quad 0 \quad 0] \end{cases}$$

In the sequel, by using Schur complement and defining: $M = M_1, M_2 = \rho_2M_1, M_3 = \rho_3M_1, M_4 = \rho_4M_1, X = M^{-1}, Y = KX^{\mathrm{T}}, \vec{P} = XPX^{\mathrm{T}}, \vec{S} = XSX^{\mathrm{T}}, \vec{T} = XTX^{\mathrm{T}}, \vec{N}_i = XN_iX^{\mathrm{T}} (i = 1,2,3,4)$, and pre-, post-multiplying both sides with $\mathrm{diag}(X \quad X \quad X \quad X \quad X \quad I \quad I \quad I)$ and its transpose, we have (19) $\Leftrightarrow \Omega < 0$, thus proof of $V'(t) < 0$.

From (18) we can see

$$x^{\mathrm{T}}(t)Qx(t) + x^{\mathrm{T}}(i_kh)K^{\mathrm{T}}RKx(i_kh) < -V'(t) \qquad (20)$$

integrating (20) from $i_kh+\tau_k$ to $i_{k+1}h+\tau_{k+1}$, and using $\bigcup_{k=1}^{\infty}[i_kh+\tau_k, i_{k+1}h+\tau_{k+1}) = [0,\infty), V(\infty) = 0$ then we get the (11). Thus we complete the proof of Theorem 1. □

Given d, in order to obtain a controller $u(t) = YX^{-\mathrm{T}}x(t)$, which achieves the least guaranteed cost value J^*, we have to solve the following minimization problem

$$\text{Minimize} \quad J_1 + J_2 + J_3 \quad \text{subject to} \quad (9) \qquad (21)$$

where

$$\begin{aligned} &J_1 = \varphi^{\mathrm{T}}(0)X^{-1}\vec{P}X^{-\mathrm{T}}\varphi(0) \\ &J_2 = \int_{-d}^{0} \varphi^{\mathrm{T}}(\alpha)X^{-1}\vec{S}X^{-\mathrm{T}}\varphi(\alpha)\mathrm{d}\alpha \\ &J_3 = \int_{0}^{\tau}\int_{-\beta}^{0} \varphi'^{\mathrm{T}}(\alpha)X^{-1}\vec{T}X^{-\mathrm{T}}\varphi'(\alpha)\mathrm{d}\alpha\mathrm{d}\beta \end{aligned} \qquad (22)$$

However, it is noted that the terms J_1, J_2, J_3 are not convex functions of X and $\vec{P}, \vec{S}, \vec{T}$. As a result, unfortunately, we can not find in general the global minimum of the above minimization problem using a convex optimization algorithm[10]. However if we can afford more computational efforts, we can obtain a guaranteed cost controller achieving a suboptimal guaranteed cost, say J_{so}^*, using an iterative algorithm presented in [11].

In the sequel, let us derive the upper bounds on the cost functions J_1, J_2, J_3. Before starting the problem, let us define matrix values $\varPi_1, \varPi_2, \varPi_3$, such that

$$\begin{aligned} &\varPi_1 = \varphi(0)\varphi^{\mathrm{T}}(0) \\ &\varPi_2 = \int_{-d}^{0} \varphi(\alpha)\varphi^{\mathrm{T}}(\alpha)\mathrm{d}\alpha \\ &\varPi_3 = \int_{0}^{\tau}\int_{-\beta}^{0} \varphi'(\alpha)\varphi'^{\mathrm{T}}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta \end{aligned} \qquad (23)$$

To derive the upper bound on J_1, let us introduce a new variable $\varLambda = \varLambda^{\mathrm{T}}$ such that

$$X^{-1}\vec{P}X^{-\mathrm{T}} < \varLambda \qquad (24)$$

By Schur complement, (24) is equivalent to

$$\begin{bmatrix} -\Lambda & X^{-1} \\ X^{-\mathrm{T}} & -\vec{P}^{-1} \end{bmatrix} < 0 \tag{25}$$

By introducing new values $M, \bar{P}$, the condition (25) can be replaced by

$$\begin{bmatrix} -\Lambda & M \\ M^{\mathrm{T}} & -\bar{P} \end{bmatrix} < 0, M = X^{-1}, \bar{P} = \vec{P}^{-1} \tag{26}$$

Assuming (26), we can conclude that the following inequality holds

$$\varphi^{\mathrm{T}}(0) X^{-1} \vec{P} X^{-\mathrm{T}} \varphi(0) < \mathrm{tr}(\Pi_1 \Lambda) \tag{27}$$

To derive the upper bound on J_2, let us introduce a new variable $\Pi = \Pi^{\mathrm{T}}$ such that

$$X^{-1} \vec{S} X^{-\mathrm{T}} < \Pi \tag{28}$$

By Schur complement, (28) is equivalent to

$$\begin{bmatrix} -\Pi & X^{-1} \\ X^{-\mathrm{T}} & -\vec{S}^{-1} \end{bmatrix} < 0 \tag{29}$$

By introducing a new value $\bar{S}$, the condition (29) can be replaced by

$$\begin{bmatrix} -\Pi & M \\ M^{\mathrm{T}} & -\bar{S} \end{bmatrix} < 0, \bar{S} = \vec{S}^{-1} \tag{30}$$

Assuming (30), we can conclude that the following inequality holds

$$\int_{-d}^{0} \varphi^{\mathrm{T}}(\alpha) X^{-1} \vec{S} X^{-\mathrm{T}} \varphi(\alpha) \mathrm{d}\alpha < \mathrm{tr}(\Pi_2 \Pi) \tag{31}$$

To derive the upper bound on J_3, let us introduce a new variable $\Xi = \Xi^{\mathrm{T}}$ such that

$$X^{-1} \vec{T} X^{-\mathrm{T}} < \Xi \tag{32}$$

By Schur complement, (32) is equivalent to

$$\begin{bmatrix} -\Xi & X^{-1} \\ X^{-\mathrm{T}} & -\vec{T}^{-1} \end{bmatrix} < 0 \tag{33}$$

By introducing a new value $\bar{T}$, the condition (33) can be replaced by

$$\begin{bmatrix} -\Xi & M \\ M^{\mathrm{T}} & -\bar{T} \end{bmatrix} < 0, \bar{T} = \vec{T}^{-1} \tag{34}$$

Assuming (34), we can conclude that the following inequality holds

$$\int_{0}^{\tau} \int_{-\beta}^{0} \varphi'^{\mathrm{T}}(\alpha) X^{-1} \vec{T} X^{-\mathrm{T}} \varphi'(\alpha) \mathrm{d}\alpha \mathrm{d}\beta < \mathrm{tr}(\Pi_3 \Xi) \tag{35}$$

For some constant J, assume

$$\mathrm{tr}(\Pi_1 \Lambda) + \mathrm{tr}(\Pi_2 \Pi) + \mathrm{tr}(\Pi_3 \Xi) < J \tag{36}$$

Combining these facts, we can construct a feasibility problem as follows

$$\begin{aligned} &\text{Find } \vec{P}, \vec{S}, \vec{T}, X, \bar{P}, \bar{S}, \bar{T}, M, \Lambda, \Pi, \Xi, Y, \vec{N}_i (i = 1, 2, 3, 4), \varepsilon \\ &\text{Subject to } \vec{P} > 0, \vec{S} > 0, \vec{T} > 0, (9)(26)(30)(34)(36) \end{aligned} \tag{37}$$

Given d and J, if the above problem has a solution, we can say that there exists a controller $u(t) = YX^{-\mathrm{T}} x(t)$ which guarantees the cost function (4) is less than J^*. Note that the conditions (26) (30) (34) still include nonlinear condition, *eg.* $\bar{P} = \vec{P}^{-1}$. However, using the idea in a cone complementary linearization algorithm[11], the above feasibility problem can be solved iteratively. Now, we suggest the following nonlinear minimization problem involving LMI conditions instead of the original non-convex minimization problem in (21)

$$\begin{aligned} &\text{Minimize } \mathrm{tr}(\vec{P}\bar{P} + \vec{S}\bar{S} + \vec{T}\bar{T} + XM) \\ &\text{Subject to } \vec{P} > 0, \vec{S} > 0, \vec{T} > 0, (9)(36) \\ &\begin{bmatrix} -\Lambda & M \\ M^{\mathrm{T}} & -\bar{P} \end{bmatrix} < 0, \begin{bmatrix} -\Pi & M \\ M^{\mathrm{T}} & -\bar{S} \end{bmatrix} < 0 \\ &\begin{bmatrix} -\Xi & M \\ M^{\mathrm{T}} & -\bar{T} \end{bmatrix} < 0, \begin{bmatrix} \vec{P} & I \\ I & \bar{P} \end{bmatrix} \geq 0 \\ &\begin{bmatrix} \vec{S} & I \\ I & \bar{S} \end{bmatrix} \geq 0, \begin{bmatrix} \vec{T} & I \\ I & \bar{T} \end{bmatrix} \geq 0 \\ &\begin{bmatrix} X & I \\ I & M \end{bmatrix} \geq 0 \end{aligned} \tag{38}$$

If the solution of the above minimization problem is $4n$, that is, $\mathrm{tr}(\vec{P}\bar{P} + \vec{S}\bar{S} + \vec{T}\bar{T} + XM) = 4n$[11], we can say from Theorem1 that the system (6) with the controller $u(t) = YX^{-\mathrm{T}} x(t)$ is uniformly asymptotically stable with the guaranteed cost J^*. Although it is still impossible to always find the globally optimal solution, the proposed nonlinear optimization problem is easier to solve than the original non-convex minimization problem in (21). Actually, utilizing the linearization method in [11], we can easily find a suboptimal minimum of the guaranteed cost using an iterative algorithm presented in the following.

Algorithm.

Step 1. Choose a sufficiently large initial J such that there exists a feasible solution in (38). Set $J_{so} = J$.

Step 2. Find a feasible set ($\vec{P}_0$, $\bar{P}_0$, $\vec{S}_0$, $\bar{S}_0$, $\vec{T}_0$, $\bar{T}_0$, X_0, M_0) satisfying LMIs in (38). Set k=1.

Step 3. Solve the following LMI problem for the variables $(\vec{P}, \bar{P}, \vec{S}, \bar{S}, \vec{T}, \bar{T}, X, M)$:

Minimize $\mathrm{tr}(\vec{P}_k \bar{P} + \vec{S}_k \bar{S} + \vec{T}_k \bar{T} + X_k M + \vec{P}\bar{P}_k + \vec{S}\bar{S}_k + \vec{T}\bar{T}_k + XM_k)$ subject to LMIs in (38).

Set $\vec{P}_{k+1} = \vec{P}, \vec{S}_{k+1} = \vec{S}, \vec{T}_{k+1} = \vec{T}, X_{k+1} = X, \bar{P}_{k+1} = \bar{P}, \bar{S}_{k+1} = \bar{S}, \bar{T}_{k+1} = \bar{T}, M_{k+1} = M$.

Step 4. If the condition (24), (28) and (32) are all satisfied, then set $J_{so} = J$ and return to Step 2 after decreasing J to some extent. If the condition (24), (28) and (32) are not satisfied within a specified number of iterations, say k_{max}, then exit. Otherwise set $k = k + 1$ and go to Step 3.

5 Numerical example

Example. Consider the following uncertain time-delay system:

$$x'(t) = \begin{bmatrix} 1 + 0.33F(t) & 0.42F(t) \\ 0.53F(t) & -2 + 0.67F(t) \end{bmatrix} x(t) +$$

$$\begin{bmatrix} 0.5 & 0.32 \\ 0 & -0.5 \end{bmatrix} x(t - d) + \begin{bmatrix} 0.47 \\ 0.75 \end{bmatrix} u(t)$$

where $d = 1, x_1(t) = 3\mathrm{e}^{t+1} - 1, x_2(t) = 0$, for $t \in [-1, 0]$

and $F^2(t) \leq 1$. We consider the cost function (11), set $Q = 0.5\mathrm{I}, R = 0.1, \rho_2 = 2, \rho_3 = 3, \rho_4 = 4$ and use the algorithm in the above, then obtain Table 1 with d given.

From Table 1: when d declines a little, the maximum allowable value τ_{max} of the network-induced increases a lot, even so, the system will be asymptotically stable as before. Accordingly, the value J_{so} drops a lot. Of course, we can give $Q, R, \rho_2, \rho_3, \rho_4$ other values, relative τ_{max}, J_{so}, K will be obtained respectively.

Table 1 When d given then obtaining τ_{max}, J_{so}, K

d	τ_{max}	J_{so}	K
1	0.15	469.98	[-2.690 -1.096]
0.5	0.45	126.63	[-1.295 0.120]

6 Conclusion

In this paper, we firstly model NCSs with network delay for a class of uncertain time-delay systems. Based on the model, a guaranteed cost controller design is presented and a sufficient condition for the existence of a guaranteed cost state-feedback for a class of uncertain time-delay systems is given and an algorithm involving a convex optimization is applied to construct a controller with a suboptimal guaranteed cost such that the system can be stabilized for all admissible uncertainties. At the same time a simulated example is given to show the effectiveness of the algorithm.

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