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Brief paper

General receding horizon control for linear time-delay systems☆

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Abstract

A general receding horizon control (RHC), or model predictive control (MPC), for time-delay systems is proposed. The proposed RHC is obtained by minimizing a new cost function that includes two terminal weighting terms, which are closely related to the closed-loop stability. The general solution of the proposed RHC is derived using the generalized Riccati method. Furthermore, an explicit solution is obtained for the case where the horizon length is less than or equal to the delay size. A linear matrix inequality (LMI) condition on the terminal weighting matrices is proposed, under which the optimal cost is guaranteed to be monotonically non-increasing. It is shown that the monotonic condition of the optimal cost guarantees closed-loop stability of the RHC. Simulations demonstrate that the proposed RHC effectively stabilizes time-delay systems.

Keywords: Receding horizon control (RHC); Cost monotonicity; Linear matrix inequality; Time-delay system

1. Introduction

In industrial processes, time-delays often occur in the transmission of material or information between different parts of a system. Chemical processing systems, transportation systems, communication systems, and power systems are typical examples of time-delay systems. Because the presence of a time-delay often causes serious deterioration of the stability and performance of the system, considerable research has been devoted to the control of time-delay systems.

For delay-free systems, the receding-horizon control (RHC), or model predictive control (MPC), has received considerable attention (Kwon & Pearson, 1977; Richalet, Rault, Testud, & Papon, 1978; Kwon & Kim, 2000) because of its many advantages, including ease of computation, good tracking performance and I/O constraint handling capability, compared with the popular steady-state infinite horizon linear quadratic (LQ) control. While the steady-state LQ control for linear systems is obtained from the algebraic Riccati equation, the RHC for linear systems is obtained from the differential or difference Riccati equation on a finite interval, which is easier to solve. Therefore, RHC has been widely used, particularly in the chemical process industries (Richalet et al., 1978).

For time-delay systems, there are no general results for the RHC. A simple control method based on the receding horizon concept has appeared in (Kwon, Jin Won Kang, Young Sam Lee, & Young Soo Moon, 2003). However, it does not have a state weighting in the cost function. Furthermore, it does not guarantee closed-loop stability by design, and therefore stability can be checked only after the controller has been designed.

In RHC, the terminal weighting matrices in the cost function are crucial for stability. The cost function taken in this paper differs from the existing forms used in (Krasovskii, 1962; Kushner & Barnea, 1970; Ross, 1971; Uchida, Shimemura, Kubo, & Abe, 1988) which consider the optimal control problem for time-delay systems. The cost function in this paper has an additional terminal weighting matrix on $x(t+T+s)$, $-h \leqslant s \leqslant 0$, where T denotes the horizon length. This additional terminal weighting matrix is necessary to guarantee closed-loop stability. For the cost function with this additional weighting matrix, we found no optimal solution in the literature. We first derive the optimal solution for this cost function using the generalized

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Riccati method taken in (Eller, Aggarwal, & Banks, 1969) and then provide the general solution of the proposed RHC using this solution. The monotonic condition of the optimal cost, under which the optimal cost is guaranteed to be monotonically nonincreasing, is proposed in terms of a linear matrix inequality on the terminal weighting matrices. Using the derived condition, we prove the stability of the proposed RHC in this paper.

2. Receding horizon control for time-delay systems

Consider a time delay system

$$\dot{x}(t) = Ax(t) + A_1 x(t-h) + Bu(t), \tag{1}$$

where the initial condition is $x(\tau) = \phi(\tau)$, $\tau \in [-h, 0]$, $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the input, and A, $A_1 \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$ are system matrices. $h > 0$ is the delay, and $\phi(t)$ is a continuous function. In order to obtain a receding horizon control, we first consider a finite horizon cost function represented by

$$\begin{aligned} J(x_{t_0}, u, t_0, t_f) = &\int_{t_0}^{t_f} [x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau)]\,\mathrm{d}\tau \\ &+ x^{\mathrm{T}}(t_f)F_1 x(t_f) \\ &+ \int_{-h}^{0} x^{\mathrm{T}}(t_f+s)F_2 x(t_f+s)\,\mathrm{d}s, \end{aligned} \tag{2}$$

where $x_{t_0} = x(t_0+s)$, $s \in [-h, 0]$, is a continuous function, t_0 is the initial time, t_f is the terminal time, and $Q \geqslant 0$, $R > 0$, $F_1 > 0$, and $F_2 > 0$. $t_f - t_0 > 0$ is the horizon length. It is noted that the cost function (2) has two terminal weighting terms. The optimal control minimizing the cost function (2) and the corresponding optimal cost will be denoted by $u^*(\tau)$, $t_0 \leqslant \tau \leqslant t_f$, and $J^*(x_{t_0}, t_0, t_f)$, respectively. It is apparent that $J^*(x_{t_0}, t_0, t_f) = J(x_{t_0}, u^*, t_0, t_f)$. The RHC is then obtained by minimizing the cost function (2) with the initial time t_0 and the terminal time t_f replaced by the current time t and $t+T$, respectively, where $T > 0$ is constant. The stability of the proposed RHC depends on the choice of terminal weighting matrices F_1 and F_2. In this section, we will first present the solution of the RHC. Stability issues will be covered in subsequent sections.

2.1. The general solution

The optimal control problem for time-delay systems with the cost function (2) with $F_2 = 0$ was considered in (Eller et al., 1969) by using the generalized Riccati method. We can obtain an RHC for the case of $F_2 = 0$ using the solution given in (Eller et al., 1969). However, it is important to note that RHC for $F_2 = 0$ does not have guaranteed stability, which will be shown in subsequent sections. In this subsection, we derive the RHC for the time-delay systems with the cost function (2) with $F_2 \neq 0$, by using the generalized Riccati method as in (Eller et al., 1969). The following notation is used.

(1) $C[-h, 0]$ is the space of functions continuous on $[-h, 0]$. By using this notation, x_τ is in $C[-h, 0]$.
(2) Let $V(\tau, x_\tau) : [t_0, t_f] \times C[-h, 0] \to \mathbf{R}$ be a continuous and differentiable functional, so define

$$\left.\frac{\mathrm{d}}{\mathrm{d}\tau} V(\tau, x_\tau)\right|_{u(\tau,x_\tau)} \triangleq \lim_{\Delta\tau\to 0}\left[\frac{V(\tau+\Delta\tau, x^u_{\tau+\Delta\tau}) - V(\tau, x_\tau)}{\Delta\tau}\right],$$

where $x^u_{\tau+\Delta\tau}(s) = x(\tau+\Delta\tau+s)$, $s \in [-h, 0]$ is the solution of system (1).

The following lemma, which is modified from that given in (Eller et al., 1969) to include the case of $F_2 \neq 0$, establishes a sufficient condition for a control u^* to be optimal.

Lemma 2.1. *If it is possible to find a continuous and differentiable functional* $V(\tau, x_\tau) : [t_0, t_f] \times C[-h, 0] \to \mathbf{R}$, *and a vector functional* $u^*(\tau, x_\tau) : [t_0, t_f] \times C[-h, 0] \to \mathbf{R}^m$ *such that*

(i) $$V(t_f, x_{t_f}) = x^{\mathrm{T}}(t_f)F_1 x(t_f) + \int_{-h}^{0} x^{\mathrm{T}}(t_f+s)F_2 x(t_f+s)\,\mathrm{d}s,$$

(ii) $$\left.\frac{\mathrm{d}}{\mathrm{d}\tau} V(\tau, x_\tau)\right|_{u^*(\tau,x_\tau)} + x^{\mathrm{T}}(\tau)Qx(\tau) + u^{*\mathrm{T}}(\tau, x_\tau)Ru^*(\tau, x_\tau) = 0,$$

(iii) $$\begin{aligned} &\left.\frac{\mathrm{d}}{\mathrm{d}\tau} V(\tau, x_\tau)\right|_{u(\tau,x_\tau)} + x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau, x_\tau)Ru(\tau, x_\tau) \\ &\quad \geqslant \left.\frac{\mathrm{d}}{\mathrm{d}\tau} V(\tau, x_\tau)\right|_{u^*(\tau,x_\tau)} + x^{\mathrm{T}}(\tau)Qx(\tau) \\ &\quad + u^{*\mathrm{T}}(\tau, x_\tau)Ru^*(\tau, x_\tau), \end{aligned}$$

for all $\tau \in [t_0, t_f]$ *and all* $x_\tau \in C[-h, 0]$. *Then* $V(t_0, x_{t_0}) = J(x_{t_0}, u^*, t_0, t_f) \leqslant J(x_{t_0}, u, t_0, t_f)$.

This lemma can be proven in a similar way to that given in (Eller et al., 1969). Therefore, the proof is omitted. This lemma states that $V(\tau, x_\tau)$ and $u^*(\tau, x_\tau)$ satisfying conditions (i) to (iii) are the optimal cost and the optimal control for the optimal control problem with the finite horizon cost function (2). The notation $u(\tau, x_\tau)$ denotes that the control law is in the form of a distributed state feedback controller.

For simplicity, the complete derivation of the optimal control using Lemma 2.1 is given in the Appendix. The optimal control $u^*(\tau)$, $t_0 \leqslant \tau \leqslant t_f$, minimizing the cost function (2) on $[t_0, t_f]$ is represented as follows:

$$u^*(\tau) = -R^{-1}B^{\mathrm{T}}\left[P_1(\tau)x(\tau) + \int_{-h}^{0} P_2(\tau,s)x(\tau+s)\,\mathrm{d}s\right],$$
$$\text{for } t_0 \leqslant \tau < t_f - h, \tag{3}$$

$$= -R^{-1}B^{\mathrm{T}}\left[W_1(\tau)x(\tau) + \int_{-h}^{t_f-\tau-h} W_2(\tau,s)x(\tau+s)\,\mathrm{d}s\right],$$
$$\text{for } t_f - h \leqslant \tau \leqslant t_f, \tag{4}$$

where $P_1(\tau)$ and $P_2(\tau,s)$ are determined as follows:

$$\dot{P}_1(\tau) - P_1(\tau)BR^{-1}B^{\mathrm{T}}P_1(\tau) + P_2(\tau,0) + P_2^{\mathrm{T}}(\tau,0) + A^{\mathrm{T}}P_1(\tau) + P_1(\tau)A + Q = 0,$$

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial s}\right)P_2(\tau,s) + A^{\mathrm{T}}P_2(\tau,s) + P_3^{\mathrm{T}}(\tau,0,s) - P_1(\tau)BR^{-1}B^{\mathrm{T}}P_2(\tau,s) = 0,$$

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial r} - \frac{\partial}{\partial s}\right)P_3(\tau,r,s) - P_2^{\mathrm{T}}(\tau,s)BR^{-1}B^{\mathrm{T}}P_2(\tau,r) = 0,$$

with boundary conditions

$$A_1^{\mathrm{T}}P_1(\tau) = P_2^{\mathrm{T}}(\tau,-h), \quad A_1^{\mathrm{T}}P_2(\tau,s) = P_3^{\mathrm{T}}(\tau,-h,s),$$

where $t_0 \leqslant \tau < t_f - h$. $W_1(\tau)$ and $W_2(\tau,s)$ are determined as follows:

$$\dot{W}_1(\tau) - W_1(\tau)BR^{-1}B^{\mathrm{T}}W_1(\tau) + A^{\mathrm{T}}W_1(\tau) + W_1(\tau)A + Q + F_2 = 0,$$

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial s}\right)W_2(\tau,s) + A^{\mathrm{T}}W_2(\tau,s) - W_1(\tau)BR^{-1}B^{\mathrm{T}}W_2(\tau,s) = 0,$$

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial r} - \frac{\partial}{\partial s}\right)W_3(\tau,r,s) - W_2^{\mathrm{T}}(\tau,s)BR^{-1}B^{\mathrm{T}}W_2(\tau,r) = 0,$$

with boundary conditions

$$A_1^{\mathrm{T}}W_1(\tau) = W_2^{\mathrm{T}}(\tau,-h), \quad A_1^{\mathrm{T}}W_2(\tau,s) = W_3^{\mathrm{T}}(\tau,-h,s),$$

where $t_f - h \leqslant \tau < t_f$. Furthermore, the following boundary conditions must be satisfied:

$$W_1(t_f) = F_1,$$

$$P_1(t_f - h) = W_1(t_f - h),$$

$$P_2(t_f - h, s) = W_2(t_f - h, s),$$

$$P_3(t_f - h, r, s) = W_3(t_f - h, r, s),$$

where $-h \leqslant s \leqslant 0$ and $-h \leqslant r \leqslant 0$.

Remark 2.1. In (Eller et al., 1969), similar types of coupled partial differential equations to the above ones were derived. The difference is that the proposed partial differential equation on $W_1(\tau)$ for $t_f - h \leqslant \tau \leqslant t_f$ has the term F_2, while the one in (Eller et al., 1969) does not. In (Eller et al., 1969), an algorithm to solve those partial differential equations was developed by transforming the original partial differential equations to ordinary differential equations. By modifying that algorithm such that it takes the difference in $W_1(\tau)$ into account, the partial differential equations given above can also be solved.

In RHC, the initial time t_0 is the current time t and the terminal time t_f is $t+T$. RHC is obtained by replacing τ with t. Depending on the horizon length T, RHC is represented in two different forms.

$$u^*(t) = -R^{-1}B^{\mathrm{T}}\left[P_1(t)x(t) + \int_{-h}^{0} P_2(t,s)x(t+s)\,\mathrm{d}s\right],$$
$$T > h, \tag{5}$$

$$= -R^{-1}B^{\mathrm{T}}\left[W_1(t)x(t) + \int_{-h}^{T-h} W_2(t,s)x(t+s)\,\mathrm{d}s\right],$$
$$T \leqslant h. \tag{6}$$

It should be noted that $P_1(t)$, $P_2(t,s)$, $W_1(t)$, and $W_2(t,s)$ are not dependent on t because the horizon length $t_f - t_0 = T$ is constant and the system is time-invariant. Therefore we can denote $P_1(t)$, $P_2(t,s)$, $W_1(t)$, and $W_2(t,s)$ by P_1, $P_2(s)$, W_1, and $W_2(s)$, respectively. Therefore, the RHC can be rewritten as follows:

$$u^*(t) = -R^{-1}B^{\mathrm{T}}\left[P_1x(t) + \int_{-h}^{0} P_2(s)x(t+s)\,\mathrm{d}s\right],$$
$$T > h, \tag{7}$$

$$= -R^{-1}B^{\mathrm{T}}\left[W_1x(t) + \int_{-h}^{T-h} W_2(s)x(t+s)\,\mathrm{d}s\right],$$
$$T \leqslant h. \tag{8}$$

2.2. *The closed-loop solution for $T \leqslant h$*

The horizon length $T = t_f - t_0$ is a design parameter and therefore can be chosen such that $T \leqslant h$. In this section, we consider the case of $T \leqslant h$, i.e., $t_f \leqslant t_0 + h$. In this case, we use the existing optimal control theory developed for delay-free systems. Assuming $t_f \leqslant t_0 + h$, the cost function

(2) can be rewritten as

$$J(x_{t_0},u,t_0,t_f)=\int_{t_0}^{t_f}[x^{\mathrm T}(\tau)\bar Q x(\tau)+u^{\mathrm T}(\tau)Ru(\tau)]\,\mathrm d\tau + x^{\mathrm T}(t_f)F_1x(t_f) + \int_{-h}^{-(t_f-t_0)} x^{\mathrm T}(t_f+s)F_2x(t_f+s)\,\mathrm ds, \quad (9)$$

where $\bar Q = Q+F_2$. It should be noted that the last integral term in the cost function (9) is constant, given x_{t_0}. Therefore, instead of the cost function in (9), we will consider the following cost function:

$$\hat J(x_{t_0},u,t_0,t_f)=\int_{t_0}^{t_f}[x^{\mathrm T}(\tau)\bar Q x(\tau)+u^{\mathrm T}(\tau)Ru(\tau)]\,\mathrm d\tau + x^{\mathrm T}(t_f)F_1x(t_f). \quad (10)$$

It is apparent that the optimal control minimizing the cost function $\hat J$ also minimizes the original cost function J. Given x_{t_0}, $x(\tau-h)$ for $t_0 \leqslant \tau \leqslant t_f$ is a known term since $t_f \leqslant t_0+h$. Therefore, we can use the existing optimal control theory (Kirk, 1970) to obtain the optimal control for the time-delay system (1). The necessary and sufficient conditions for optimal control are as follows:

$$\dot x(\tau)=Ax(\tau)+A_1x(\tau-h)+Bu(\tau), \quad (11)$$

$$\dot p(\tau)=-\bar Q x(\tau)-A^{\mathrm T}p(\tau), \quad (12)$$

$$0=Ru(\tau)+B^{\mathrm T}p(\tau), \quad (13)$$

$$p(t_f)=F_1x(t_f), \quad (14)$$

where $t_0 \leqslant \tau \leqslant t_f$. From (13) the optimal control is represented by

$$u(\tau)=-R^{-1}B^{\mathrm T}p(\tau), \qquad t_0 \leqslant \tau \leqslant t_f. \quad (15)$$

Substituting (15) into (11) yields

$$\dot x(\tau)=Ax(\tau)+A_1x(\tau-h)-BR^{-1}B^{\mathrm T}p(\tau).$$

Therefore, we have the set of $2n$ linear differential equations

$$\begin{bmatrix}\dot x(\tau)\\ \dot p(\tau)\end{bmatrix}=\begin{bmatrix}A & -BR^{-1}B^{\mathrm T}\\ -\bar Q & -A^{\mathrm T}\end{bmatrix}\begin{bmatrix}x(\tau)\\ p(\tau)\end{bmatrix}+\begin{bmatrix}A_1\\ 0\end{bmatrix}x(\tau-h). \quad (16)$$

Define

$$\mathbf H \triangleq \begin{bmatrix}A & -BR^{-1}B^{\mathrm T}\\ -\bar Q & -A^{\mathrm T}\end{bmatrix},$$

$$\Phi(\tau) \triangleq \mathrm e^{\mathbf H\tau}=\begin{bmatrix}\Phi_{11}(\tau) & \Phi_{12}(\tau)\\ \Phi_{21}(\tau) & \Phi_{22}(\tau)\end{bmatrix}.$$

Let $\Phi(\tau)=\mathrm e^{\mathbf H\tau}$. Because $t_f \leqslant t_0+h$, $x(\tau-h)$ is a known term for $t_0 \leqslant \tau \leqslant t_f$. Therefore, using linear system theory, we obtain

$$\begin{bmatrix}x(t_f)\\ p(t_f)\end{bmatrix}=\begin{bmatrix}\Phi_{11}(t_f-\tau) & \Phi_{12}(t_f-\tau)\\ \Phi_{21}(t_f-\tau) & \Phi_{22}(t_f-\tau)\end{bmatrix}\begin{bmatrix}x(\tau)\\ p(\tau)\end{bmatrix} + \int_{-h}^{t_f-\tau-h}\begin{bmatrix}\Phi_{11}(t_f-\tau-s-h)A_1x(\tau+s)\\ \Phi_{21}(t_f-\tau-s-h)A_1x(\tau+s)\end{bmatrix}\mathrm ds. \quad (17)$$

From the boundary conditions (14) and (17) we obtain

$$p(\tau)=W_1(\tau)x(\tau)+\int_{-h}^{t_f-\tau-h}W_2(\tau,s)x(\tau+s)\,\mathrm ds, \quad (18)$$

where

$$W_1(\tau)=[\Phi_{22}(t_f-\tau)-F_1\Phi_{12}(t_f-\tau)]^{-1}\times[F_1\Phi_{11}(t_f-\tau)-\Phi_{21}(t_f-\tau)],$$

$$W_2(\tau,s)=[\Phi_{22}(t_f-\tau)-F_1\Phi_{12}(t_f-\tau)]^{-1}\times[F_1\Phi_{11}(t_f-\tau-s-h)-\Phi_{21}(t_f-\tau-s-h)]A_1.$$

The RHC is obtained by replacing τ by t, t_0 by t, and t_f by $t+T$, respectively, as follows:

$$u^*(t)=-R^{-1}B^{\mathrm T}\left[W_1(t)x(t)+\int_{-h}^{T-h}W_2(t,s)x(t+s)\,\mathrm ds\right].$$

Note that $W_1(t)$ and $W_2(t,s)$ are independent of t. Therefore, we can denote $W_1(t)$ and $W_2(t,s)$ by W_1 and $W_2(s)$, respectively. Therefore, RHC can be written in the simpler form as

$$u^*(t)=-R^{-1}B^{\mathrm T}\left[W_1x(t)+\int_{-h}^{T-h}W_2(s)x(t+s)\,\mathrm ds\right], \quad (19)$$

where

$$W_1=[\Phi_{22}(T)-F_1\Phi_{12}(T)]^{-1}[F_1\Phi_{11}(T)-\Phi_{21}(T)],$$

$$W_2(s)=[\Phi_{22}(T)-F_1\Phi_{12}(T)]^{-1}[F_1\Phi_{11}(T-s-h)-\Phi_{21}(T-s-h)]A_1.$$

However, the above formula is not recommended numerically because $\mathbf H$ has unstable eigenvalues. Using the idea in (Lewis & Syroms, 1995), we can obtain the explicit representation for W_1 and $W_2(s)$ for the case where the eigenvalues of $\mathbf H$ are not on the imaginary axis.

The matrix $\mathbf H$, whose eigenvalues are not on the imaginary axis, is decomposed as

$$\mathbf H = WDW^{-1},$$

where

$$D=\begin{bmatrix}-M & 0\\ 0 & M\end{bmatrix},\quad W=\begin{bmatrix}W_{11} & W_{12}\\ W_{21} & W_{22}\end{bmatrix}.$$

M is a diagonal matrix containing the unstable eigenvalues of $\mathbf{H}$. The columns of W are eigenvectors of the corresponding eigenvalues. Denote W^{-1} by

$$W^{-1}=\begin{bmatrix}\hat{W}_{11} & \hat{W}_{12}\\ \hat{W}_{21} & \hat{W}_{22}\end{bmatrix}.$$

We can then represent W_1 and $W_2(s)$ explicitly as

$$W_1=[-\bar{W}_1\hat{W}_{12}+\hat{W}_{22}]^{-1}[\bar{W}_1\hat{W}_{11}-\hat{W}_{21}],$$

$$W_2(s)=[-\bar{W}_1\hat{W}_{12}+\hat{W}_{22}]^{-1}\mathrm{e}^{-M(s+h)}\times[\mathrm{e}^{-M(T-s-h)}\bar{W}_1(T)\mathrm{e}^{-M(T-s-h)}\hat{W}_{11}-\hat{W}_{21}]A_1,$$

where

$$\bar{W}_1(T)=[W_{22}-F_1W_{12}]^{-1}[F_1W_{11}-W_{21}],$$

$$\bar{W}_1=\mathrm{e}^{-MT}\bar{W}_1(T)\mathrm{e}^{-MT}.$$

3. Monotonic condition of the optimal cost

In this section, we present a condition on the terminal weighting matrices F_1 and F_2, under which the optimal cost $J^*(x_{t_0},t_0,\sigma)$ does not increase as the terminal time σ increases. We will call it a monotonic condition of the optimal cost on the terminal weighting matrices.

Theorem 3.1. *Assume that F_1 and F_2 in (2) satisfy the following matrix inequality for some K_1 and K_2:*

$$\begin{bmatrix}P_{11} & F_1(A_1+BK_2)+K_1^{\mathrm{T}}RK_2\\ (A_1+BK_2)^{\mathrm{T}}F_1+K_2^{\mathrm{T}}RK_1 & K_2^{\mathrm{T}}RK_2-F_2\end{bmatrix}\leqslant 0, \tag{20}$$

where $P_{11}=(A+BK_1)^{\mathrm{T}}F_1+F_1(A+BK_1)+Q+K_1^{\mathrm{T}}RK_1+F_2$. The optimal cost $J^*(x_{t_0},t_0,\sigma)$ then satisfies the following relation:

$$\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}\leqslant 0. \tag{21}$$

Proof.

$$\begin{aligned}\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}&=\lim_{\Delta\to 0}\frac{1}{\Delta}\{J^*(x_{t_0},t_0,\sigma+\Delta)-J^*(x_{t_0},t_0,\sigma)\},\\ &=\lim_{\Delta\to 0}\frac{1}{\Delta}\left\{\int_{t_0}^{\sigma+\Delta}[\bar{x}^{\mathrm{T}}(\tau)Q\bar{x}(\tau)+\bar{u}^{\mathrm{T}}(\tau)R\bar{u}(\tau)]\,\mathrm{d}\tau\right.\\ &\quad+\bar{x}^{\mathrm{T}}(\sigma+\Delta)F_1\bar{x}(\sigma+\Delta)\\ &\quad+\int_{-h}^{0}\bar{x}^{\mathrm{T}}(\sigma+\Delta+s)F_2\bar{x}(\sigma+\Delta+s)\,\mathrm{d}s\\ &\quad-\int_{t_0}^{\sigma}[\hat{x}^{\mathrm{T}}(\tau)Q\hat{x}(\tau)+\hat{u}^{\mathrm{T}}(\tau)R\hat{u}(\tau)]\,\mathrm{d}\tau\\ &\quad-\hat{x}^{\mathrm{T}}(\sigma)F_1\hat{x}(\sigma)\\ &\quad\left.-\int_{-h}^{0}\hat{x}^{\mathrm{T}}(\sigma+s)F_2\hat{x}(\sigma+s)\,\mathrm{d}s\right\},\end{aligned}$$

where $\bar{u}(\cdot)$ and $\hat{u}(\cdot)$ are the optimal controls which minimize the cost function J for the terminal time $\sigma+\Delta$ and σ, respectively. $\bar{x}(\cdot)$ and $\hat{x}(\cdot)$ are state trajectories generated when the system is controlled by $\bar{u}(\cdot)$ and $\hat{u}(\cdot)$, respectively. Assume that the control $\bar{u}(\tau)$ is replaced by $\tilde{u}(\tau)$, which is given by

$$\tilde{u}(\tau)=\begin{cases}\hat{u}(\tau), & t_0\leqslant\tau<\sigma\\ K_1\hat{x}(\tau)+K_2\hat{x}(\tau-h), & \sigma\leqslant\tau\leqslant\sigma+\Delta.\end{cases} \tag{22}$$

It is noted that $\hat{x}(\tau)$ for $\tau\in[\sigma,\sigma+\Delta]$ is the state trajectory resulting from $\tilde{u}(\tau)$. $\tilde{u}(\tau)$ is not an optimal control in case that the terminal time is $\sigma+\Delta$. Therefore, we have

$$\begin{aligned}\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}&\leqslant\lim_{\Delta\to 0}\frac{1}{\Delta}\left\{\int_{\sigma}^{\sigma+\Delta}[\hat{x}^{\mathrm{T}}(\tau)Q\hat{x}(\tau)+(K_1\hat{x}(\tau)+K_2\hat{x}(\tau-h))^{\mathrm{T}}\right.\\ &\quad\times R(K_1\hat{x}(\tau)+K_2\hat{x}(\tau-h))]\,\mathrm{d}\tau+\hat{x}^{\mathrm{T}}(\sigma+\Delta)F_1\hat{x}(\sigma+\Delta)\\ &\quad+\int_{-h}^{0}\hat{x}^{\mathrm{T}}(\sigma+\Delta+s)F_2\hat{x}(\sigma+\Delta+s)\,\mathrm{d}s\\ &\quad\left.-\hat{x}^{\mathrm{T}}(\sigma)F_1\hat{x}(\sigma)-\int_{-h}^{0}\hat{x}^{\mathrm{T}}(\sigma+s)F_2\hat{x}(\sigma+s)\,\mathrm{d}s\right\},\\ &=\hat{x}^{\mathrm{T}}(\sigma)Q\hat{x}(\sigma)+(K_1\hat{x}(\sigma)+K_2\hat{x}(\sigma-h))^{\mathrm{T}}R\\ &\quad\times(K_1\hat{x}(\sigma)+K_2\hat{x}(\sigma-h))+\frac{\mathrm{d}}{\mathrm{d}\sigma}\hat{x}^{\mathrm{T}}(\sigma)F_1\hat{x}(\sigma)\\ &\quad+\frac{\mathrm{d}}{\mathrm{d}\sigma}\int_{-h}^{0}\hat{x}^{\mathrm{T}}(\sigma+s)F_2\hat{x}(\sigma+s)\,\mathrm{d}s,\\ &=\begin{bmatrix}\hat{x}(\sigma)\\ \hat{x}(\sigma-h)\end{bmatrix}^{\mathrm{T}}\Lambda\begin{bmatrix}\hat{x}(\sigma)\\ \hat{x}(\sigma-h)\end{bmatrix},\end{aligned}$$

where Λ is the matrix in (20). The matrix Λ is independent of both t_0 and σ. Therefore, it is clear that $\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}\leqslant 0$ for any $\sigma>t_0$ if $\Lambda\leqslant 0$. This completes the proof. □

Remark 3.1. It is noted that the case of $F_2=0$ corresponds to a conventional cost function chosen to derive the RHC for delay-free systems. In that case, however, inequality (20) is not feasible. Adding the last term with nonzero F_2 in (2) makes inequality (20) more likely to be feasible.

Remark 3.2. The last segment of control (22) is arbitrary and chosen to obtain the cost monotonicity condition (21). There are multiple choices. We choose the control form $K_1x(\tau)+K_2x(\tau-h)$ in order to represent a sufficient condition for stability on terminal weighting matrices F_1 and F_2 in terms of LMI for easy computation.

Recently numerous useful algorithms to solve linear matrix inequality problems have been developed. If the inequality condition (20) can be represented in an LMI form, we will be able to obtain the solution easily. Therefore, the matrix inequality (20) in Theorem 3.1 is converted to an LMI as follows:

Theorem 3.2. *Assume that there exist $X>0$, S, Y_1, and Y_2 such that*

$$\begin{bmatrix} (AX+BY_1)^{\mathrm{T}}+(AX+BY_1) & (A_1S+BY_2) & XQ^{\frac{1}{2}} & Y_1^{\mathrm{T}}R^{\frac{1}{2}} & X \\ (A_1S+BY_2)^{\mathrm{T}} & -S & 0 & Y_2^{\mathrm{T}}R^{\frac{1}{2}} & 0 \\ Q^{\frac{1}{2}}X & 0 & -I & 0 & 0 \\ R^{\frac{1}{2}}Y_1 & R^{\frac{1}{2}}Y_2 & 0 & -I & 0 \\ X & 0 & 0 & 0 & -S \end{bmatrix} \leqslant 0. \tag{23}$$

Then $F_1=X^{-1}$, $F_2=S^{-1}$, $K_1=Y_1X^{-1}$, and $K_2=Y_2S^{-1}$ satisfy the matrix inequality (20) *of Theorem* 3.1.

Proof. The inequality condition (20) can be rearranged as

$$\begin{bmatrix} (A+BK_1)^{\mathrm{T}}F_1+F_1(A+BK_1) & F_1(A_1+BK_2) \\ (A_1+BK_2)^{\mathrm{T}}F_1 & -F_2 \end{bmatrix} + \begin{bmatrix} Q^{\frac{1}{2}} & K_1^{\mathrm{T}}R^{\frac{1}{2}} & I \\ 0 & K_2^{\mathrm{T}}R^{\frac{1}{2}} & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & F_2 \end{bmatrix} \times \begin{bmatrix} Q^{\frac{1}{2}} & 0 \\ R^{\frac{1}{2}}K_1 & R^{\frac{1}{2}}K_2 \\ I & 0 \end{bmatrix} \leqslant 0. \tag{24}$$

By introducing change of variables such that

$$X=F_1^{-1}, \quad S=F_2^{-1}, \quad Y_1=K_1F_1^{-1}, \quad Y_2=K_2F_2^{-1},$$

and using the Schur complement, the inequality condition (24) is equivalently changed into the LMI form of (23). This completes the proof. □

4. Stability of RHC

This section investigates the stability of the RHC.

Theorem 4.1. *Given $Q>0$ and $R>0$, if $\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}\leqslant 0$ for $\sigma>t_0$, system* (1) *controlled by the RHC is asymptotically stable.*

Proof. If $\frac{\partial J^*(x_{t_0},t_0,\sigma)}{\partial\sigma}\leqslant 0$ for $\sigma>t_0$, we have

$$\begin{aligned} J^*(x_t,t,t+T) &= \int_t^{t+\theta}[x^{\mathrm{T}}(\tau)Qx(\tau)+u^{*\mathrm{T}}(\tau)Ru^*(\tau)]\,\mathrm{d}\tau \\ &\quad + J^*(x_{t+\theta},t+\theta,t+T), \\ &\geqslant \int_t^{t+\theta}[x^{\mathrm{T}}(\tau)Qx(\tau)+u^{*\mathrm{T}}(\tau)Ru^*(\tau)]\,\mathrm{d}\tau \\ &\quad + J^*(x_{t+\theta},t+\theta,t+T+\theta). \end{aligned}$$

Rearranging the above inequality and dividing it by θ yield

$$\frac{J^*(x_{t+\theta},t+\theta,t+T+\theta)-J^*(x_t,t,t+T)}{\theta} \leqslant -\frac{1}{\theta}\int_t^{t+\theta}[x^{\mathrm{T}}(\tau)Qx(\tau)+u^{*\mathrm{T}}(\tau)Ru^*(\tau)]\,\mathrm{d}\tau.$$

If $\theta\to 0$, we obtain

$$\frac{\mathrm{d}J^*(x_t,t,t+T)}{\mathrm{d}t}\leqslant -[x^{\mathrm{T}}(t)Qx(t)+u^{*\mathrm{T}}(t)Ru^*(t)],$$

which shows that $J^*(x_t,t,t+T)$ is nonincreasing. Because $J^*(x_t,t,t+T)\geqslant 0$, $J^*(x_t,t,t+T)\to c$ and $\frac{\mathrm{d}J^*(x_t,t,t+T)}{\mathrm{d}t}\to 0$ as $t\to\infty$, where c is a nonnegative constant. From this, it is clear that $x(t)\to 0$ and $u^*(t)\to 0$ as $t\to\infty$. Therefore the closed-loop system is asymptotically stable. This completes the proof. □

Using the sufficient conditions for monotonic cost, the following theorem is obtained.

Theorem 4.2. *Given $Q>0$ and $R>0$, if the matrix inequality* (20) *or the LMI* (23) *is feasible, system* (1) *is asymptotically stabilizable with the proposed RHC.*

5. Numerical examples

In this section, a numerical example is presented to illustrate the proposed methods.

Example 5.1. Consider a time-delay system whose system matrices are given by

$$A=\begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix}, \quad A_1=\begin{bmatrix} 0 & 0 \\ -1 & -0.5 \end{bmatrix}, \quad B=\begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

The delay size of the system is $h=1$. It is noted that this system is open-loop unstable. The weighting matrices Q and R are chosen such that $Q=I$ and $R=I$. Terminal weighting matrices F_1 and F_2 guaranteeing the closed-loop stability

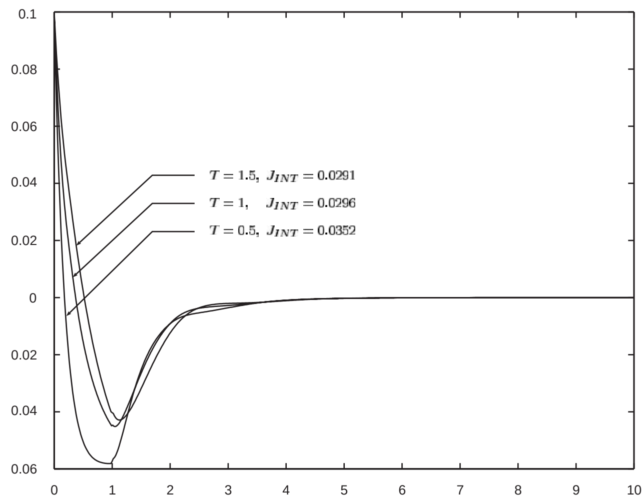

Fig. 1. State trajectories of x_2 due to RHCs with different horizon lengths.

are obtained by solving the LMI (23)

$$F_1 = \begin{bmatrix} 4.7880 & 1.9737 \\ 1.9737 & 7.1028 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1.0799 & 0.0831 \\ 0.0831 & 0.9533 \end{bmatrix}.$$

We design three RHCs with the horizon lengths, $T = 0.5,\ 1$, and 1.5 and apply them to the system with initial state $\phi_1(\theta) = 0.2,\ \phi_2(\theta) = 0.1,\ -1 \leqslant \theta \leqslant 0$. In order to measure the performances of the designed controllers, we define an integrated cost

$$J_{\mathrm{INT}} = \int_0^{10} [x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)]\,\mathrm{d}t.$$

Fig. 1 compares the state trajectories of x_2 resulting from those three RHCs. Note that J_{INT} decreases as T increases. In the case of $T = 1$, the explicit RHC is given by (19) with W_1 and $W_2(s)$

$$W_1 = \begin{bmatrix} 1.1469 & 0.0444 \\ 0.0444 & 0.7552 \end{bmatrix},$$

$$W_2(s) = \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \end{bmatrix},$$

where

$$W_{11}(s) = -0.016322\mathrm{e}^{5.2545s} + 0.13599\mathrm{e}^{1.8885s} - 0.22032\mathrm{e}^{-5.2545(s+1)} + 0.0423\mathrm{e}^{-1.8885(s+1)},$$

$$W_{12}(s) = -0.0081609\mathrm{e}^{5.2545s} + 0.067996\mathrm{e}^{1.8885s} - 0.11016\mathrm{e}^{-5.2545(s+1)} + 0.02115\mathrm{e}^{-1.8885(s+1)},$$

$$W_{21}(s) = 0.0056925\mathrm{e}^{5.2545s} - 0.058528\mathrm{e}^{1.8885s} - 0.68129\mathrm{e}^{-5.2545(s+1)} - 0.018451\mathrm{e}^{-1.8885(s+1)},$$

$$W_{22}(s) = 0.0028463\mathrm{e}^{5.2545s} - 0.029264\mathrm{e}^{1.8885s} - 0.34064\mathrm{e}^{-5.2545(s+1)} - 0.0092257\mathrm{e}^{-1.8885(s+1)}.$$

In case of $T = 1.5$, the horizon length is longer than the delay size, h. In this case, the coupled partial differential equations should be solved to obtain the RHC given in (7). As already stated in Remark 2.1, we use the algorithm presented in (Eller et al., 1969) to solve the partial differential equations. The step size for the numerical integration was chosen to be 0.01.

From this example, we see that the proposed RHC stabilizes time-delay systems. Furthermore, by adjusting the horizon length, we can change the controller performance.

6. Conclusions

A stabilizing RHC (or MPC) for linear time-delay systems is proposed. We propose a new receding horizon cost function and have derived the RHC minimizing that cost function. The general solution of the proposed RHC was derived using the generalized Riccati method. Furthermore, the explicit solution was obtained for the case where the horizon length is less than or equal to the delay. A linear matrix inequality condition on the terminal weighting matrix for the RHC, which guarantees that the optimal cost is monotonic for the time-delay system, was proposed. Under that condition, it has been shown that the closed-loop stability of the RHC is guaranteed. The proposed RHC for time-delay systems in this paper is given for the first time in a general form. The solution of the proposed RHC is obtained more easily than the conventional infinite-horizon LQ control, for which coupled algebraic Riccati equation should be solved. Furthermore, the proposed RHC has a merit that it can take the performance criterion into account, while some of other stabilizing state-feedback controls for time-delay systems can only achieve stability. It can be widely used for various processes, particularly for chemical processes. Extension of the proposed results to time-delay systems with input constraint is considered to be an interesting future work.

Appendix

As in (Eller et al., 1969), two continuous functions, $V_1(\tau, x_\tau)$ and $V_2(\tau, x_\tau)$, are chosen to avoid discontinuity and satisfy Condition (i) of Lemma 2.1. Using $V_1(\tau, x_\tau)$ and $V_2(\tau, x_\tau)$, the optimal controls $u_1(\cdot)$ and $u_2(\cdot)$ are also obtained for each horizon, i.e., $t_0 \leqslant \tau < t_f - h$ and $t_f - h \leqslant \tau \leqslant t_f$.

Case I ($t_0 \leqslant \tau < t_f - h$):

The proof for Case I is exactly the same as that given in (Eller et al., 1969). See (Eller et al., 1969) for the form of $V_1(\tau, x_\tau)$ and the complete proof.

Case II ($t_f - h \leqslant \tau \leqslant t_f$):
Define

$$
\begin{aligned}
V_2(\tau, x_\tau) &= x^{\mathrm{T}}(\tau)W_1(\tau)x(\tau) + 2x^{\mathrm{T}}(\tau)\int_{-h}^{t_f-\tau-h} W_2(\tau,s)x(\tau+s)\,\mathrm{d}s \\
&\quad + \int_{-h}^{t_f-\tau-h}\int_{-h}^{t_f-\tau-h} x^{\mathrm{T}}(\tau+s)W_3(\tau,r,s)x(\tau+r)\,\mathrm{d}r\,\mathrm{d}s \\
&\quad + \int_{t_f-\tau-h}^{0} x^{\mathrm{T}}(\tau+s)F_2x(\tau+s)\,\mathrm{d}s. \qquad \text{(A.1)}
\end{aligned}
$$

Using the relations

$$\int_{t_f-\tau-h}^{0} x^{\mathrm{T}}(\tau+s)F_2x(\tau+s)\,\mathrm{d}s = \int_{t_f-h}^{\tau} x^{\mathrm{T}}(\alpha)F_2x(\alpha)\,\mathrm{d}\alpha,$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_{t_f-h}^{\tau} x^{\mathrm{T}}(\alpha)F_2x(\alpha)\,\mathrm{d}\alpha = x^{\mathrm{T}}(\tau)F_2x(\tau),$$

and proceeding as in Case I of (Eller et al., 1969) produce the following set of conditions:

$$
\begin{aligned}
&\dot{W}_1(\tau) - W_1(\tau)BR^{-1}B^{\mathrm{T}}W_1(\tau) + A^{\mathrm{T}}W_1(\tau) \\
&\quad + W_1(\tau)A + Q + F_2 = 0, \\
&\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial s}\right)W_2(\tau,s) + A^{\mathrm{T}}W_2(\tau,s) \\
&\quad - W_1(\tau)BR^{-1}B^{\mathrm{T}}W_2(\tau,s) = 0, \\
&\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial r} - \frac{\partial}{\partial s}\right)W_3(\tau,r,s) \\
&\quad - W_2^{\mathrm{T}}(\tau,s)BR^{-1}B^{\mathrm{T}}W_2(\tau,r) = 0,
\end{aligned}
$$

with boundary condition

$$A_1^{\mathrm{T}}W_1(\tau) = W_2^{\mathrm{T}}(\tau,-h),$$

$$A_1^{\mathrm{T}}W_2(\tau,s) = W_3^{\mathrm{T}}(\tau,-h,s).$$

The optimal control $u_2(\tau)$ is

$$
\begin{aligned}
u_2(\tau) &= -R^{-1}B^{\mathrm{T}}W_1(\tau)x(\tau) \\
&\quad - R^{-1}B^{\mathrm{T}}\int_{-h}^{t_f-\tau-h} W_2(\tau,s)x(\tau+s)\,\mathrm{d}s,
\end{aligned}
$$

where $t_f - h \leqslant \tau \leqslant t_f$. Because $V_1(\tau,x_\tau)$ is the optimal cost function for $t_0 \leqslant \tau < t_f - h$ and $V_2(\tau,x_\tau)$ is the optimal cost function for $t_f - h \leqslant \tau \leqslant t_f$, it is clear that $V_1(t_f-h, x_{t_f-h}) = V_2(t_f-h, x_{t_f-h})$ or

$$
\begin{aligned}
&x^{\mathrm{T}}(t_f-h)[P_1(t_f-h) - W_1(t_f-h)]x(t_f-h) \\
&\quad + x^{\mathrm{T}}(t_f-h)\int_{-h}^{0}[P_2(t_f-h,s) - W_2(t_f-h,s)]\,\mathrm{d}s \\
&\quad + \int_{-h}^{0}\int_{-h}^{0} x^{\mathrm{T}}(t_f-h+s)[P_3(t_f-h,r,s) \\
&\quad - W_3(t_f-h,r,s)]x(t_f-h+r)\,\mathrm{d}r\,\mathrm{d}s = 0. \qquad \text{(A.2)}
\end{aligned}
$$

must be satisfied. Eq. (A.2) is satisfied if and only if

$$P_1(t_f-h) = W_1(t_f-h),$$

$$P_2(t_f-h,s) = W_2(t_f-h,s),$$

$$P_3(t_f-h,r,s) = W_3(t_f-h,r,s).$$

Further, $W_1(t_f-h)$, $W_2(t_f-h)$ and $W_3(t_f-h,r,s)$ are used as additional boundary conditions. $V_1(\tau,x_\tau)$, $V_2(\tau,x_\tau)$, $u_1(\tau)$ and $u_2(\tau)$ are now combined as follows:

$$
\begin{aligned}
u^*(\tau) &= u_1(\tau), \quad t_0 \leqslant \tau < t_f - h, \\
&= u_2(\tau), \quad t_f - h \leqslant \tau \leqslant t_f,
\end{aligned}
$$

$$
\begin{aligned}
V(\tau,x_\tau) &= V_1(\tau,x_\tau), \quad t_0 \leqslant \tau < t_f - h, \\
&= V_2(\tau,x_\tau), \quad t_f - h \leqslant \tau \leqslant t_f.
\end{aligned}
$$

Therefore, $u^*(\tau)$ and $V(\tau,x_\tau)$ defined in this way satisfy Conditions (2) and (3) of Lemma 2.1. All that remains is to satisfy Condition (1) of Lemma 2.1. To do this, set

$$W_1(t_f) = F_1.$$

Thus, $V(\tau,x_\tau)$ and $u^*(\tau)$ satisfy Lemma 2.1 and therefore are the optimal cost and the optimal control, respectively. Note that $V(\tau,x_\tau)$ is both continuous and differentiable at time $\tau = t_f - h$ from boundary conditions so that Lemma 2.1 is well applied on $[t_0, t_f]$.

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