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Delay-dependent robust stabilization of uncertain state-delayed systems

YOUNG SOO MOON†, POOGYEON PARK‡, WOOK HYUN KWON†* and YOUNG SAM LEE†

This paper concerns a problem of robust stabilization of uncertain state-delayed systems. A new delay-dependent stabilization condition using a memoryless controller is formulated in terms of matrix inequalities. An algorithm involving convex optimization is proposed to design a controller guaranteeing a suboptimal maximal delay such that the system can be stabilized for all admissible uncertainties.

1. Introduction

During the last decades, considerable attention has been devoted to the problem of stability analysis and controller design for time-delay systems. Especially, in accordance with the advance of robust control theory, a number of robust stabilization methods have been proposed for uncertain time-delay systems.

The existing robust stabilization results for time-delay systems can be classified into two types: delay-independent stabilization (Phoojaruenchanachai and Furuta 1992, Xie and de Souza 1993, Mahmoud and Al-Muthairi 1994, Lee *et al.* 1994, Kim *et al.* 1996) and delay-dependent stabilization (Niculescu *et al.* 1994, Li and de Souza 1997 a, b, Fu *et al.* 1998, Li *et al.* 1998). The delay-independent stabilization provides a controller which can stabilize a system irrespective of the size of the delay. On the other hand, the delay-dependent stabilization is concerned with the size of the delay and usually provides an upper bound of the delay such that the closed-loop system is stable for any delay less than the upper bound. While the delay-independent stabilization has been extensively studied by many researchers for the last decades, the study for the delay-dependent stabilization is relatively new and still under progress (Niculescu *et al.* 1998). In general, the delay-dependent stabilization is considered less conservative than the delay-independent case. However, the existing delay-dependent stabilization results are still too conservative in some cases. In particular, when applied to a system which is stabilizable independent of the size of the delay, the existing delay-dependent stabilization methods often yield very conservative results, far from providing infinity as the upper bound of the allowable delay.

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* Author for correspondence. e-mail: whkwon@cisl.snu.ac.kr

† School of Electrical Engineering and ERC-ACI, Seoul National University, Seoul, 151-742, Korea.

‡ Department of Electronic and Electrical Engineering, Pohang University of Science and Technology, Pohang, Kyung-Book, 790-784, Korea.

In this paper, a new delay-dependent robust stabilization condition using a memoryless controller is presented for uncertain state-delayed systems. An algorithm involving convex optimization is proposed to design a controller guaranteeing a suboptimal maximal delay such that the system can be stabilized for all admissible uncertainties. It is shown by numerical examples that the proposed stabilization method can be less conservative than existing results and even capture the delay-independent stabilizability of the system, which is not possible with the previous results. It is also shown that the conservatism can be further reduced using a delayed feedback control for a case where the size of the delay is known.

The organization of the paper is as follows. In § 2, the problem to be solved is formulated and preliminary results are given. In § 3, nominal state-delayed systems without uncertainties are considered first and stability analysis and stabilization results are presented. Then, § 4 deals with uncertain time-delay systems and the results of the previous section are extended to robust stability and stabilization conditions. In § 5, numerical examples are given for a comparison of the proposed method with previous results and finally § 6 makes conclusions.

2. Problem statement and preliminaries

Consider the following uncertain state-delayed systems

$$\left.\begin{aligned} \dot{x}(t) &= (A + DF(t)E)x(t) + (A_1 + D_1F_1(t)E_1)x(t-h) \\ &\quad + (B + DF(t)E_b)u(t) \\ x(t) &= \phi(t), \quad t \in [-h, 0] \end{aligned}\right\} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control, $h > 0$ is the delay of the system, $\phi(\cdot)$ is the initial condition, A, A_1, B, D, D_1, E, E_1 and E_b are real constant matrices with appropriate dimensions and $F(t) \in R^{j \times k}$ and $F_1(t) \in R^{j_1 \times k_1}$ are time-varying uncertainties satisfying

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$$\|F(t)\| \leq 1, \qquad \|F_1(t)\| \leq 1$$

We are interested in designing a memoryless state-feedback controller

$$u(t) = Gx(t) \tag{2}$$

where $G \in R^{m\times n}$ is a constant gain matrix. Our aim is to develop a delay-dependent robust stabilization method which provides a controller gain G as well as an upper bound $\bar{h}$ of the delay such that the closed-loop system is stable for any h satisfying $0 \leq h \leq \bar{h}$ and for all admissible uncertainties.

For a special case where the information on the size of the delay h is available, we also consider a delayed feedback controller of the form

$$u(t) = Gx(t) + G_1 x(t-h) \tag{3}$$

Although a memoryless controller (2) has an advantage of easy implementation, its performance cannot be better than a delayed feedback controller which utilize the available information of the size of the delay. A more general form of a delayed feedback controller might be

$$u(t) = Gx(t) + \int_{t-h}^{t} G_2(s)x(s)\,\mathrm{d}s \tag{4}$$

However, the task of storing all the previous states $x(\cdot)$ and computing the values of the time-varying gain matrices $G_2(\cdot)$ makes the practical realization of the infinite-dimensional controller (4) very difficult. In this respect, the controller (3) could be considered as a compromise between the performance improvement and the implementational simplicity. For the controller given by (4), see Ross (1971).

In obtaining the main results of this paper, the following upper bound for the inner product of two vectors plays an important role

$$-2a^{\mathrm{T}}b \leq \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-I \\ Y^{\mathrm{T}}-I & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{5}$$

where

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$$

and I denotes an identity matrix with an appropriate dimension.

A special choice of Y and Z such that $Y = I$ and $Z = X^{-1}$ in (5) provides a well-known upper bound

$$-2a^{\mathrm{T}}b \leq \inf_{X>0} \{a^{\mathrm{T}}Xa + b^{\mathrm{T}}X^{-1}b\}$$

Also, choosing $Y = I + XM$ and $Z = (M^{\mathrm{T}}X + I)X^{-1} \times (XM + I)$ gives an upper bound

$$-2a^{\mathrm{T}}b \leq \inf_{X>0,M} \{(a+Mb)^{\mathrm{T}}X(a+Mb) + b^{\mathrm{T}}X^{-1}b + 2b^{\mathrm{T}}Mb\} \tag{6}$$

which was introduced by the authors in Park *et al.* (1998) and Park (1999) for stability analysis of time-delay systems. Although the underlying idea is similar, the upper bound (5) has a simpler form than (6), which enables one to solve synthesis problems.

Extending the idea of (5), we have the following lemma.

Lemma 1: *Assume that $a(\cdot) \in \mathcal{R}^{n_a}$, $b(\cdot) \in \mathcal{R}^{n_b}$ and $\mathcal{N}(\cdot) \in \mathcal{R}^{n_a\times n_b}$ are defined on the interval Ω. Then, for any matrices $X \in \mathcal{R}^{n_a\times n_a}$, $Y \in \mathcal{R}^{n_a\times n_b}$ and $Z \in \mathcal{R}^{n_b\times n_b}$, the following holds*

$$-2\int_{\Omega} a^{\mathrm{T}}(\alpha)\mathcal{N}b(\alpha)\,\mathrm{d}\alpha \leq \int_{\Omega} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^{\mathrm{T}} \times \begin{bmatrix} X & Y-\mathcal{N} \\ Y^{\mathrm{T}}-\mathcal{N}^{\mathrm{T}} & Z \end{bmatrix} \times \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} \mathrm{d}\alpha \tag{7}$$

where

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$$

In the following section, instead of directly dealing with the uncertain system (1), we first consider a nominal system without uncertainties and present stability and stabilization conditions.

3. Stability and stabilization for nominal systems

Let us consider a nominal state-delayed system

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-h) + Bu(t) \\ x(t) &= \phi(t), \quad t \in [-h, 0] \end{aligned}\right\} \tag{8}$$

We start with stability analysis of the unforced system (8) with $u(t) = 0$. The following theorem presents a delay-dependent stability condition, which is the starting point of our further developments.

Theorem 1: *If there exist $P > 0$, $Q > 0$, X, Y and Z such that*

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + \bar{h}X + Y + Y^{\mathrm{T}} + Q & -Y + PA_1 & \bar{h}A^{\mathrm{T}}Z \\ -Y^{\mathrm{T}} + A_1^{\mathrm{T}}P & -Q & \bar{h}A_1^{\mathrm{T}}Z \\ \bar{h}ZA & \bar{h}ZA_1 & -\bar{h}Z \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0 \tag{10}$$

then the unforced system (8) *with* $u(t) = 0$ *is asymptotically stable for any time-delay* h *satisfying* $0 \le h \le \bar{h}$.

Proof: Choose a Lyapunov functional as

$$V(x(t-\alpha), \alpha \in [0, \bar{h}]) = V_1 + V_2 + V_3 \qquad (11)$$

where

$$V_1 \triangleq x^{\mathrm{T}}(t)Px(t)$$

$$V_2 \triangleq \int_{-h}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\beta$$

$$V_3 \triangleq \int_{t-h}^{t} x^{\mathrm{T}}(\alpha)Qx(\alpha)\,\mathrm{d}\alpha$$

Since it holds that

$$x(t) - x(t-h) \equiv \int_{t-h}^{t} \dot{x}(\sigma)\,\mathrm{d}\sigma = \int_{t-h}^{t} [Ax(\sigma) + A_1x(\sigma - h)]\,\mathrm{d}\sigma$$

the unforced system (8) can be written as (Hale and Lunel 1993)

$$\dot{x}(t) = (A + A_1)x(t) - A_1\int_{t-h}^{t} [Ax(\alpha) + A_1x(\alpha - h)]\,\mathrm{d}\alpha$$

and thus the derivative of V_1 satisfies the relation

$$\dot{V}_1 = 2x^{\mathrm{T}}(t)P(A + A_1)x(t) - 2x^{\mathrm{T}}(t)PA_1\int_{t-h}^{t} \dot{x}(\alpha)\,\mathrm{d}\alpha$$

Defining $a(\cdot)$, $b(\cdot)$, and $\mathcal{N}$ in (7) as $a(\alpha) \triangleq x(t)$, $b(\alpha) \triangleq \dot{x}(\alpha)$, and $\mathcal{N} \triangleq PA_1$ for all $\alpha \in [t-h, t]$ and applying Lemma 1 will supply (10) and

$$\begin{aligned}\dot{V}_1 &\le 2x^{\mathrm{T}}(t)P(A + A_1)x(t) + hx^{\mathrm{T}}(t)Xx(t) \\ &\quad + 2x^{\mathrm{T}}(t)(Y - PA_1)\int_{t-h}^{t} \dot{x}(\alpha)\,\mathrm{d}\alpha \\ &\quad + \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &\le x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA + \bar{h}X + Y + Y^{\mathrm{T}})x(t) \\ &\quad + 2x^{\mathrm{T}}(t)(PA_1 - Y)x(t-h) + \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\,\mathrm{d}\alpha\end{aligned}$$

Since $\dot{V}_2$ and $\dot{V}_3$ yield the relation

$$\dot{V}_2 = h[Ax(t) + A_1x(t-h)]^{\mathrm{T}}Z[Ax(t) + A_1x(t-h)] - \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z\dot{x}(\alpha)\,\mathrm{d}\alpha$$

$$\dot{V}_3 = x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-h)Qx(t-h)$$

we have the derivative of V as

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\ &\le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^{\mathrm{T}} \\ &\quad\times \begin{bmatrix} (1,1) & PA_1 - Y + \bar{h}A^{\mathrm{T}}ZA_1 \\ A_1^{\mathrm{T}}P - Y^{\mathrm{T}} + \bar{h}A_1^{\mathrm{T}}ZA & -Q + \bar{h}A_1^{\mathrm{T}}ZA_1 \end{bmatrix} \\ &\quad\times \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}\end{aligned}$$

where

$$(1,1) \triangleq A^{\mathrm{T}}P + PA + \bar{h}X + Y + Y^{\mathrm{T}} + \bar{h}A^{\mathrm{T}}ZA + Q$$

Then, using the Lyapunov–Krasovskii stability theorem (Hale and Lunel 1993) and Schur complement (Boyd *et al.* 1994), we can conclude that the unforced system (8) is asymptotically stable if (9) and (10) hold. This completes the proof. □

The proposed stability conditions (9) and (10) are linear matrix inequality (LMI) conditions. Hence, it is easy to compute the maximum upper bound of the allowable delay $\bar{h}$ using efficient convex optimization algorithms (Boyd *et al.* 1994).

In the following theorem, we extend Theorem 1 to design a stabilizing memoryless controller (2) for the system (8).

Theorem 2: *If there exist* $L > 0$, M, N, R, V *and* $W > 0$ *such that*

$$\begin{bmatrix} (1,1) & -N + A_1L & \bar{h}(LA^{\mathrm{T}} + V^{\mathrm{T}}B^{\mathrm{T}}) \\ -N^{\mathrm{T}} + LA_1^{\mathrm{T}} & -W & \bar{h}LA_1^{\mathrm{T}} \\ \bar{h}(AL + BV) & \bar{h}A_1L & -\bar{h}R \end{bmatrix} < 0 \qquad (12)$$

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & LR^{-1}L \end{bmatrix} \ge 0 \qquad (13)$$

where

$$(1,1) \triangleq LA^{\mathrm{T}} + AL + BV + V^{\mathrm{T}}B^{\mathrm{T}} + \bar{h}M + N + N^{\mathrm{T}} + W$$

then the system (8) *with the control* $u(t) = VL^{-1}x(t)$ *is asymptotically stable for any time-delay* h *satisfying* $0 \le h \le \bar{h}$.

Proof: In view of the closed-loop system of (8) with the control (2), we replace A in (9) with $A + BG$. Now, pre- and postmultiply $\mathrm{diag}\,(P^{-1}, P^{-1}, Z^{-1})$ and $\mathrm{diag}\,(P^{-1}, P^{-1})$ to (9) and (10), respectively and apply the change of variables such that $L \triangleq P^{-1}$, $M \triangleq P^{-1}XP^{-1}$, $N \triangleq P^{-1}YP^{-1}$, $R \triangleq Z^{-1}$, $W \triangleq P^{-1}QP^{-1}$, and $V \triangleq GP^{-1}$, then we obtain (12) and (13). This completes the proof. □

It is noted that the resulting conditions for synthesis problem in Theorem 2 are no more LMI conditions because of the term $LR^{-1}L$ in (13). As a result, unfortunately, we cannot find in general the global maximum $\bar{h}$ using convex optimization algorithms in this case.

An easy way to obtain a suboptimal maximal delay instead is simply setting $R = L$ in (12) and (13), which results in LMI conditions. However, if one can afford more computational efforts, better results can be obtained using an iterative algorithm presented next.

First, we define a new variable S such that $LR^{-1}L \geq S$ and replace the condition (13) with

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & S \end{bmatrix} \geq 0, \quad LR^{-1}L \geq S \tag{14}$$

Since $LR^{-1}L \geq S$ is equivalent to $L^{-1}RL^{-1} \leq S^{-1}$, the condition (14) is equal to

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & S \end{bmatrix} \geq 0, \quad \begin{bmatrix} S^{-1} & L^{-1} \\ L^{-1} & R^{-1} \end{bmatrix} \geq 0$$

by Schur complement (Boyd *et al.* 1994). Then, by introducing new variables T, J and K, the original condition (13) can be represented as

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & S \end{bmatrix} \geq 0, \quad \begin{bmatrix} T & J \\ J & K \end{bmatrix} \geq 0,$$

$$T = S^{-1}, \quad J = L^{-1}, \quad K = R^{-1}$$

Now, using a cone complementarity problem (El Ghaoui *et al.* 1997), we suggest the following non-linear minimization problem involving LMI conditions instead of the original non-convex feasibility problem of Theorem 2

$$\left.\begin{array}{ll} \text{Minimize} & \mathrm{Tr}(ST + LJ + RK) \\ \text{subject to} & \text{(12) and} \\ & \begin{bmatrix} M & N \\ N^{\mathrm{T}} & S \end{bmatrix} \geq 0, \quad \begin{bmatrix} T & J \\ J & K \end{bmatrix} \geq 0 \\ & L > 0, \quad W > 0 \\ & \begin{bmatrix} S & I \\ I & T \end{bmatrix} \geq 0, \quad \begin{bmatrix} L & I \\ I & J \end{bmatrix} \geq 0 \\ & \begin{bmatrix} R & I \\ I & K \end{bmatrix} \geq 0 \end{array}\right\} \tag{15}$$

If the solution of the above minimization problem is $3n$, that is, $\mathrm{Tr}\,(ST + LJ + RK) = 3n$, we can say from Theorem 2 that the system (8) with the control $u(t) = VL^{-1}x(t)$ is asymptotically stable. Although it is still impossible to always find the global optimal solution, the proposed non-linear minimization problem is easier to solve than the original non-convex feasibility problem. Actually, utilizing the linearization method (El Ghaoui *et al.* 1997), we can easily find a suboptimal maximal delay using an iterative algorithm presented in the following. Note that the condition (13) is used as a stopping criterion in the algorithm since it is numerically very difficult in practice to obtain the optimal solution such that $\mathrm{Tr}\,(ST + LJ + RK)$ is exactly equal to $3n$.

Algorithm 1:

Step 1. Choose a sufficiently small initial $\bar{h} > 0$ such that there exists a feasible solution to (12) and (15). Set $\bar{h}_{so} = \bar{h}$.

Step 2. Find a feasible set $(J_0, K_0, L_0, M_0, N_0, R_0, S_0, T_0, V_0, W_0)$ satisfying (12) and (15). Set $k = 0$.

Step 3. Solve the following LMI problem for the variables $(J, K, L, M, N, R, S, T, V, W)$

$$\begin{array}{ll} \text{Minimize} & \mathrm{Tr}\,(S_k T + T_k S + L_k J + J_k L + R_k K + K_k R) \\ \text{subject to} & \text{(12) and (15)} \end{array}$$

Set $J_{k+1} = J$, $K_{k+1} = K$, $L_{k+1} = L$, $R_{k+1} = R$, $S_{k+1} = S$ and $T_{k+1} = T$.

Step 4. If the condition (13) is satisfied, then set $\bar{h}_{so} = \bar{h}$ and return to Step 2 after increasing $\bar{h}$ to some extent. If the condition (13) is not satisfied within a specified number of iterations, say $k_{\max}$, then exit. Otherwise, set $k = k + 1$ and go to Step 3.

The above algorithm gives a suboptimal maximal delay $\bar{h}_{so}$ such that the system (8) can be stabilized with the controller (2). Later, in § 5, we shall illustrate via numerical examples that the above algorithm can provide quite satisfactory results.

Now, let us consider a special case where the size of the delay h is known. In this case, the delayed feedback controller (3) can be used instead of the memoryless controller (2). By setting $A_1 \triangleq A_1 + BG_1$ in (12) and applying the change of variable $V_1 \triangleq G_1 P^{-1}$, we can obtain a stabilization condition for a delayed feedback controller (3) as follows.

Corollary 1: *Suppose that h is known. If there exist $L > 0$, M, N, R, V, V_1 and $W > 0$ such that*

$$\begin{bmatrix} (1,1) & -N + A_1 L + BV_1 & h(LA^{\mathrm{T}} + V^{\mathrm{T}}B^{\mathrm{T}}) \\ -N^{\mathrm{T}} + LA_1^{\mathrm{T}} + V_1^{\mathrm{T}}B^{\mathrm{T}} & -W & h(LA_1^{\mathrm{T}} + V_1^{\mathrm{T}}B^{\mathrm{T}}) \\ h(AL + BV) & h(A_1 L + BV_1) & -hR \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & LR^{-1}L \end{bmatrix} \geq 0 \tag{17}$$

where

$$(1,1) \triangleq LA^{\mathrm{T}} + AL + BV + V^{\mathrm{T}}B^{\mathrm{T}} + hM + N + N^{\mathrm{T}} + W$$

then the system (8) *with the control*

$$u(t) = VL^{-1}x(t) + V_1 L^{-1} x(t-h)$$

is asymptotically stable.

Algorithm 1 can be similarly applied to obtain a suboptimal maximal delay h for a delayed feedback controller.

In the following section, we extend the obtained stability and stabilization conditions to robust conditions for the uncertain system (1).

4. Robust stability and stabilization for uncertain systems

The following theorem provides robust stability analysis of the unforced system (1) with $u(t) = 0$.

Theorem 3: *If there exist matrices $P > 0$, $Q > 0$, X, Y, Z and scalars e_1 and e_2 such that*

$$\begin{bmatrix} Y_{11} & -Y + PA_1 & \bar{h}A^{\mathrm{T}}Z & PD & PD_1 \\ -Y^{\mathrm{T}} + A_1^{\mathrm{T}}P & -Q + e_2 E_1^{\mathrm{T}} E_1 & \bar{h}A_1^{\mathrm{T}}Z & 0 & 0 \\ \bar{h}ZA & \bar{h}ZA_1 & -\bar{h}Z & \bar{h}ZD & \bar{h}ZD_1 \\ D^{\mathrm{T}}P & 0 & \bar{h}D^{\mathrm{T}}Z & -e_1 I & 0 \\ D_1^{\mathrm{T}}P & 0 & \bar{h}D_1^{\mathrm{T}}Z & 0 & -e_2 I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0 \tag{19}$$

where

$$Y_{11} \triangleq A^{\mathrm{T}}P + PA + \bar{h}X + Y + Y^{\mathrm{T}} + Q + e_1 E^{\mathrm{T}} E$$

then the unforced system (1) *with $u(t) = 0$ is asymptotically stable for any time-delay h satisfying $0 \leq h \leq \bar{h}$ and all admissible uncertainties.*

Proof: Replace A and A_1 in (9) with $A + DF(t)E$ and $A_1 + D_1 F_1(t) E_1$, respectively and multiply both sides of the resulting matrix by vectors x_i for $i = 1, \ldots, 3$. Next, define

$$p \triangleq F(t)Ex_1, \quad q \triangleq F_1(t)E_1 x_2$$

Then we have the condition

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ p \\ q \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_{11} & -Y + PA_1 & \bar{h}A^{\mathrm{T}}Z & PD & PD_1 \\ -Y^{\mathrm{T}} + A_1^{\mathrm{T}}P & -Q & \bar{h}A_1^{\mathrm{T}}Z & 0 & 0 \\ \bar{h}ZA & \bar{h}ZA_1 & -\bar{h}Z & \bar{h}ZD & \bar{h}ZD_1 \\ D^{\mathrm{T}}P & 0 & \bar{h}D^{\mathrm{T}}Z & 0 & 0 \\ D_1^{\mathrm{T}}P & 0 & \bar{h}D_1^{\mathrm{T}}Z & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ p \\ q \end{bmatrix} < 0 \tag{20}$$

for all admissible $F(t)$ and $F_1(t)$, where

$$X_{11} \triangleq A^{\mathrm{T}}P + PA + \bar{h}X + Y + Y^{\mathrm{T}} + Q$$

Since the conditions $\|F(t)\| \leq 1$ and $\|F_1(t)\| \leq 1$ can be replaced with the existence conditions of $e_1 > 0$ and $e_2 > 0$ such that

$$e_1 p^{\mathrm{T}} p \leq e_1 x_1^{\mathrm{T}} E^{\mathrm{T}} E x_1, \qquad e_2 q^{\mathrm{T}} q \leq e_2 x_3^{\mathrm{T}} E_1^{\mathrm{T}} E_1 x_3$$

applying the S-procedure (Boyd *et al.* 1994) allows us to obtain (18). This completes the proof. □

Next, we extend Theorem 3 to design a robust stabilizing memoryless controller (2) for the system (1) in the following theorem.

Theorem 4: *If there exist matrices $L > 0$, M, N, R, V, $W > 0$ and scalars $e_1, e_2, \ldots, e_6$ such that*

$$\begin{bmatrix} Y_{11} & -N + A_1 L & Y_{13} & LE^{\mathrm{T}} + V^{\mathrm{T}} E_b^{\mathrm{T}} & Y_{15} & 0 & 0 \\ -N^{\mathrm{T}} + LA_1^{\mathrm{T}} & -W & \bar{h}LA_1^{\mathrm{T}} & 0 & 0 & LE_1^{\mathrm{T}} & \bar{h}LE_1^{\mathrm{T}} \\ Y_{13}^{\mathrm{T}} & \bar{h}A_1 L & Y_{33} & 0 & 0 & 0 & 0 \\ EL + E_b V & 0 & 0 & -e_1 I & -e_3 I & 0 & 0 \\ Y_{15}^{\mathrm{T}} & 0 & 0 & -e_3 I & -e_2 I & 0 & 0 \\ 0 & E_1 L & 0 & 0 & 0 & -e_4 I & -e_6 I \\ 0 & \bar{h}E_1 L & 0 & 0 & 0 & -e_6 I & -e_5 I \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} M & N \\ N^{\mathrm{T}} & LR^{-1}L \end{bmatrix} \geq 0 \tag{22}$$

where

$$\begin{aligned} Y_{11} &\triangleq LA^{\mathrm{T}} + AL + BV + V^{\mathrm{T}}B^{\mathrm{T}} + \bar{h}M \\ &\quad + N + N^{\mathrm{T}} + W + e_1 DD^{\mathrm{T}} + e_4 D_1 D_1^{\mathrm{T}} \\ Y_{13} &\triangleq \bar{h}(LA^{\mathrm{T}} + V^{\mathrm{T}}B^{\mathrm{T}}) + e_3 DD^{\mathrm{T}} + e_6 D_1 D_1^{\mathrm{T}} \\ Y_{15} &\triangleq \bar{h}(LE^{\mathrm{T}} + V^{\mathrm{T}}E_b^{\mathrm{T}}) \\ Y_{33} &\triangleq -\bar{h}R + e_2 DD^{\mathrm{T}} + e_5 D_1 D_1^{\mathrm{T}} \end{aligned}$$

then the system (1) *with the control $u(t) = VL^{-1}x(t)$ is asymptotically stable for any time-delay h satisfying $0 \leq h \leq \bar{h}$ and all admissible uncertainties.*

Proof: As in the proof of Theorem 3, replace A, A_1 and B in (12) with $A + DF(t)E$, $A_1 + D_1 F_1(t) E_1$ and $B + DF(t)E_b$, respectively and multiply both sides of the resulting matrix by vectors x_i for $i = 1, \ldots, 3$. If we define

$$p_1 \triangleq F^{\mathrm{T}}(t)D^{\mathrm{T}}x_1, \quad p_2 \triangleq F^{\mathrm{T}}(t)D^{\mathrm{T}}x_3$$

$$q_1 \triangleq F_1^{\mathrm{T}}(t)D_1^{\mathrm{T}}x_1, \quad q_2 \triangleq F_1^{\mathrm{T}}(t)D_1^{\mathrm{T}}x_3$$

then we have the following condition

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ p_1 \\ p_2 \\ q_1 \\ q_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X_{11} & -N + A_1 L & X_{13} & X_{14} & X_{15} & 0 & 0 \\ -N^{\mathrm{T}} + LA_1^{\mathrm{T}} & -W & \bar{h}LA_1^{\mathrm{T}} & 0 & 0 & LE_1^{\mathrm{T}} & \bar{h}LE_1^{\mathrm{T}} \\ X_{13}^{\mathrm{T}} & \bar{h}A_1 L & -\bar{h}R & 0 & 0 & 0 & 0 \\ X_{14}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{15}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_1 L & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{h}E_1 L & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ p_1 \\ p_2 \\ q_1 \\ q_2 \end{bmatrix} < 0 \tag{23}$$

for all admissible $F(t)$ and $F_1(t)$, where

$$
\begin{aligned}
X_{11} &\triangleq LA^{\mathrm{T}} + AL + BV + V^{\mathrm{T}}B^{\mathrm{T}} + \bar{h}M + N + N^{\mathrm{T}} + W \\
X_{13} &\triangleq \bar{h}(LA^{\mathrm{T}} + V^{\mathrm{T}}B^{\mathrm{T}}) \\
X_{14} &\triangleq LE^{\mathrm{T}} + V^{\mathrm{T}}E_b^{\mathrm{T}} \\
X_{15} &\triangleq \bar{h}(LE^{\mathrm{T}} + V^{\mathrm{T}}E_b^{\mathrm{T}})
\end{aligned}
$$

We shall now claim that the condition $\|F(t)\| \le 1$ can be replaced with the condition that there exist e_1, e_2, e_3 such that

$$
\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \le \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix} \tag{24}
$$

$$
\begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} > 0 \tag{25}
$$

To prove this claim, we first UDL-decompose the left-hand side of (25) into

$$
\begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} = \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix} \begin{bmatrix} g_1 I & 0 \\ 0 & g_2 I \end{bmatrix} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix}^{\mathrm{T}}
$$

where g_1 and g_2 are positive because the UDL-decomposition preserves matrix inertia. Now consider the left-hand side of (24)

$$
\begin{aligned}
&\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \\
&= \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix} \begin{bmatrix} g_1 I & 0 \\ 0 & g_2 I \end{bmatrix} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \\
&= \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix} \begin{bmatrix} g_1 F(t)F^{\mathrm{T}}(t) & 0 \\ 0 & g_2 F(t)F^{\mathrm{T}}(t) \end{bmatrix} \\
&\quad \times \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix} \\
&\le \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix} \begin{bmatrix} g_1 I & 0 \\ 0 & g_2 I \end{bmatrix} \begin{bmatrix} I & f_1 I \\ 0 & I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix} \\
&= \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_1 I & e_3 I \\ e_3 I & e_2 I \end{bmatrix} \begin{bmatrix} D^{\mathrm{T}}x_1 \\ D^{\mathrm{T}}x_3 \end{bmatrix}
\end{aligned}
$$

Similarly, $\|F_1(t)\| \le 1$ can be replaced with the condition that there exist e_4, e_5, e_6 such that

$$
\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_4 I & e_6 I \\ e_6 I & e_5 I \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \le \begin{bmatrix} D_1^{\mathrm{T}}x_1 \\ D_1^{\mathrm{T}}x_3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} e_4 I & e_6 I \\ e_6 I & e_5 I \end{bmatrix} \begin{bmatrix} D_1^{\mathrm{T}}x_1 \\ D_1^{\mathrm{T}}x_3 \end{bmatrix} \tag{26}
$$

$$
\begin{bmatrix} e_4 I & e_6 I \\ e_6 I & e_5 I \end{bmatrix} > 0 \tag{27}
$$

Now applying the S-procedure (Boyd *et al.* 1994) to (23), (24), (25), (26) and (27), we can obtain (21). This completes the proof. $\square$

If we consider a case where the size of the delay h is measurable, a stabilization condition for a delayed feedback controller (3) can be obtained as follows.

Corollary 2: *Suppose that h is known. If there exist matrices $L > 0$, M, N, R, V, V_1, $W > 0$ and scalars $e_1, e_2, \ldots, e_6$ such that*

$$
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} & LE^{\mathrm{T}} + V^{\mathrm{T}}E_b^{\mathrm{T}} & h(LE^{\mathrm{T}} + V^{\mathrm{T}}E_b^{\mathrm{T}}) & 0 & 0 \\
Y_{12}^{\mathrm{T}} & -W & Y_{23} & V_1^{\mathrm{T}}E_b^{\mathrm{T}} & hV_1^{\mathrm{T}}E_b^{\mathrm{T}} & LE_1^{\mathrm{T}} & hLE_1^{\mathrm{T}} \\
Y_{13}^{\mathrm{T}} & Y_{23}^{\mathrm{T}} & Y_{33} & 0 & 0 & 0 & 0 \\
EL + E_b V & E_b V_1 & 0 & -e_1 I & -e_3 I & 0 & 0 \\
h(EL + E_b V) & hE_b V_1 & 0 & -e_3 I & -e_2 I & 0 & 0 \\
0 & E_1 L & 0 & 0 & 0 & -e_4 I & -e_6 I \\
0 & hE_1 L & 0 & 0 & 0 & -e_6 I & -e_5 I
\end{bmatrix} < 0
$$

$$
\begin{bmatrix} M & N \\ N^{\mathrm{T}} & LR^{-1}L \end{bmatrix} \ge 0
$$

where

$$
\begin{aligned}
Y_{11} &\triangleq LA^{\mathrm{T}} + AL + BV + V^{\mathrm{T}}B^{\mathrm{T}} + hM + N + N^{\mathrm{T}} \\
&\quad + W + e_1 DD^{\mathrm{T}} + e_4 D_1 D_1^{\mathrm{T}} \\
Y_{12} &\triangleq -N + A_1 L + BV_1 \\
Y_{13} &\triangleq h(LA^{\mathrm{T}} + V^{\mathrm{T}}B^{\mathrm{T}}) + e_3 DD^{\mathrm{T}} + e_6 D_1 D_1^{\mathrm{T}} \\
Y_{23} &\triangleq h(LA_1^{\mathrm{T}} + V_1^{\mathrm{T}}B^{\mathrm{T}}) \\
Y_{33} &\triangleq -hR + e_2 DD^{\mathrm{T}} + e_5 D_1 D_1^{\mathrm{T}}
\end{aligned}
$$

then the system (1) *with the control*

$$
u(t) = VL^{-1}x(t) + V_1 L^{-1} x(t - h)
$$

is asymptotically stable.

To compute a suboptimal maximal value of the allowable delay $\bar{h}$ (or h) in Theorem 4 (or Corollary 2), Algorithm 1 for nominal systems can be similarly used since the conditions with the non-convex term $LR^{-1}L$ are the same as in the nominal case of the previous section.

The following section presents numerical examples which compare the proposed stabilization methods with previous results.

5. Numerical examples

Let us consider the uncertain state-delayed system (1) with system matrices

$$
A = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
$$

and uncertainties

$$D = D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad E = E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_b = 0$$

The above system is delay-independently stabilizable. That is to say, the above system is robust stabilizable for any h satisfying $0 \le h < \infty$. With the existing delay-dependent robust stabilization results (Niculescu *et al.* 1994, Li and de Souza 1997 a, b), however, one can only obtain quite conservative results. In fact, the largest upper bound of the delay among existing results is given by Li and de Souza (1997 a) as $\bar{h} = 0.5557$. On the other hand, the delay-dependent stabilization conditions proposed in Theorem 4 using Algorithm 1 provides $\bar{h}_{so} = \infty$ as expected.

The next example considers the case where the system is not delay-independently stabilizable. Let us consider the uncertain time-delay system (1) with system matrices

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{28}$$

and the same uncertainties as in the above example. In this case, the largest time-delay attainable from the known delay-dependent robust stabilization methods in the literature (Niculescu *et al.* 1994, Li and de Souza 1997 a, b) is $\bar{h} = 0.3015$ (Li and de Souza 1997 a). In the following, we shall illustrate that the stabilization conditions proposed in this paper provides less conservative results.

First, if we use the LMI conditions which can be obtained from Theorem 4 by setting $L = R$ in (21) and (22), we get $\bar{h} = 0.3830$ with a stabilizing controller

$$u(t) = [\,-0.8226 \quad -3.0988\,]x(t)$$

Using Algorithm 1, we can obtain better results as shown in table 1.

Note that the number of iterations in table 1 denotes after how many iterations the stopping criterion, i.e. the condition (13), was activated. From table 1, it is clear that the robust stabilizing controller suggested in this paper can be less conservative than the existing results. For instance, even for $\bar{h}_{so} = 0.4500$, a stabilizing controller can be obtained as

$$u(t) = [\,-4.8122 \quad -7.7129\,]x(t)$$

Now, we consider a special case where the size of the delay h is known. In this case, table 2 shows that the delayed feedback controller suggested in Corollary 2 using Algorithm 1 allows larger delays than memoryless controllers.

The advantage of a delayed feedback controller over a memoryless controller can be more clearly shown for the nominal system (28) without uncertainties. In this nominal case, the largest allowable delay among the existing results is $\bar{h} = 0.4999$ (Li and de Souza 1997 a) using a memoryless controller. Using the memoryless controller of Theorem 4 in this paper, we obtain $\bar{h}_{so} = 1.0000$ after 60 iterations. On the other hand, if we suppose that the delay h is measurable, then the delayed feedback control (3) can stabilize the system for much larger values of delays. For instance, even when $h = 1000$, a stabilizing controller can be obtained as

$$u(t) = [-0.00599 \quad -1.00149]x(t) + [7.99783 \quad 2.99945]x(t - 1000)$$

$\bar{h}_{so}$	Number of iterations
0.3700	1
0.3750	2
0.4000	55
0.4500	99

Table 1. Memoryless controller of Theorem 4.

h	Number of iterations
0.3700	1
0.3800	2
0.5000	28
0.5700	80

Table 2. Delayed feedback controller of Corollary 2.

The last example considers a liquid monopropellant rocket motor with a pressure feeding system, which is more practical and complex than the previous examples. This system is not delay-independently stabilizable either. Through this example, we will illustrate the robustness of the proposed stabilization method. A linearized version of the feeding system and combustion chamber equations, assuming nonsteady flow, is given by Fiagbedzi and Pearson (1986)

$$\left.\begin{aligned} \dot{\phi}(t) &= (\gamma - 1)\phi(t) - \gamma\phi(t-h) + \mu(t-h) \\ \dot{\mu}_1(t) &= \frac{1}{\xi J}\left[-\psi(t) + \frac{p_0 - p_1}{2\Delta p}\right] \\ \dot{\mu}(t) &= \frac{1}{(1-\xi)J}[-\mu(t) + \psi(t) - P\phi(t)] \\ \dot{\psi}(t) &= \frac{1}{E}[\mu_1(t) - \mu(t)] \end{aligned}\right\} \tag{29}$$

where t is the reduced time normalized by the gas residence time θ_g in steady operation, $h = \bar{\tau}/\theta_g$ is the reduced time lag with $\bar{\tau}$ the value of the time lag in steady operations, $\phi(t) = [p(t) - \bar{p}]/\bar{p}$, with $p(t)$ the instantaneous pressure in the combustion chamber and $\bar{p}$ the pressure in the combustion chamber in steady operation, $\mu(t) = (\dot{m}_i - \bar{\dot{m}})/\bar{\dot{m}}$, with $\dot{m}_i$ the instantaneous mass rate of injected propellant and $\bar{\dot{m}}$ the value of $\dot{m}_i$ in steady operation, $\mu_1(t) = [\dot{m}_1(t) - \bar{\dot{m}}]/\bar{\dot{m}}$, with $\dot{m}_1(t)$ the instantaneous mass flow upstream of the capacitance, $\psi(t) = [p_1(t) - \bar{p}_1]/(2\Delta p)$, with $p_1(t)$ the instantaneous pressure at the place in the feeding line where the capacitance representing the elasticity is located, $\bar{p}_1$ the value of p_1 in steady operation and $\Delta p = \bar{p}_1 - \bar{p}$ the injector pressure drop in steady operation, p_0 is the regulated gas pressure for the pressure supply, $P = \bar{p}/(2\Delta p)$, γ is the pressure exponent of the pressure dependence of the combustion process taking place during the time lag, ξ represents the fractional length for the pressure supply, J is the inertia parameter of the line, and E is the elasticity parameter of the line. Guided by the Fiagbedzi and Pearson (1986), we take $u = (p_0 - p_1)/(2\Delta p)$ as a control variable and adopt the following representative numerical values: $\xi = 0.5, P = 1, J = 2$ and $E = 1$. We assume parameter uncertainty on γ, which is represented by $\gamma(t) = 1 + \alpha\delta(t)$, where $\alpha > 0$ and $|\delta(t)| \le 1$. Hence, $1 - \alpha \le \gamma(t) \le 1 + \alpha$. Letting

$$x(t) = [\phi(t) \quad \mu_1(t) \quad \mu(t) \quad \psi(t)]^{\mathrm{T}}$$

the system (29) reduces to (1), where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D = D_1 = [\alpha \ \ 0 \ \ 0 \ \ 0]^{\mathrm{T}}$$

$$E = [1 \ \ 0 \ \ 0 \ \ 0], \qquad E_1 = [-1 \ \ 0 \ \ 0 \ \ 0]$$

$$E_b = 0, \quad F(t) = F_1(t) = \delta(t)$$

In case that $\alpha = 0.15$ and $\bar{h} = 1$, we have obtained the following controller from Theorem 4

$$u(t) = [976.9055 \quad -205.2514 \quad 817.4449 \quad -1011.8588]x(t) \tag{30}$$

According to Theorem 4, the closed-loop system should remain asymptotically stable against all admissible uncertainties represented by $0.85 \le \gamma(t) \le 1.15$ and $h \le 1$. Figures 1 and 2 show the state trajectories of $x_1(t)$ when controller (30) is applied. Initial condition is assumed to be $x(t) = [1\ 1\ 1\ 1]^{\mathrm{T}}$, $t \in [-h, 0]$. Figure 1 show that the controller (30) works well if $h \le 1$, while the system is somewhat unstable when $h = 1.3$. Figure 2 shows that the controller (30) works well if $0.85 \le \gamma \le 1.15$ and the system oscillates when $\gamma = 1.25$. From this example, it is clearly demonstrated that the controller obtained from Theorem 4 robustly stabilizes the uncertain time-delay systems.

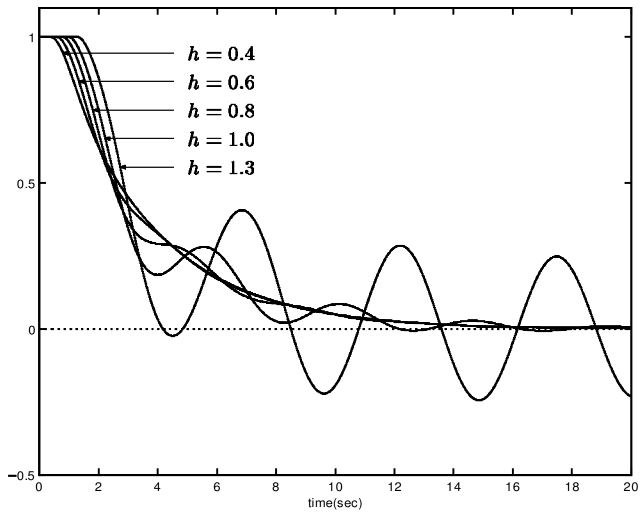

Figure 1. Variation of $x_1(t)$ with h.

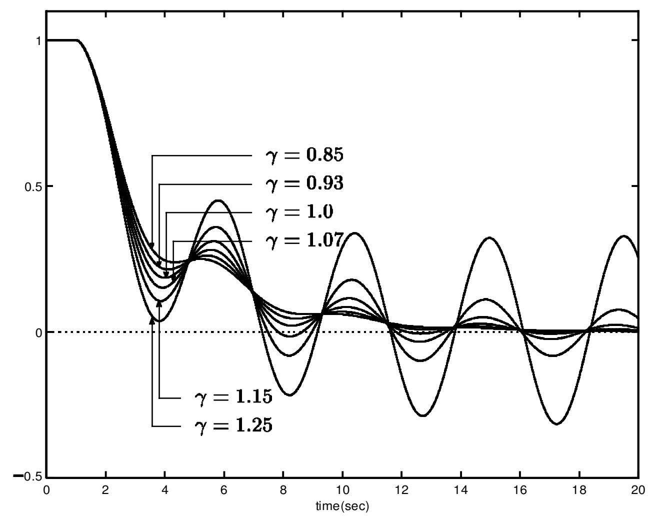

Figure 2. Variation of $x_1(t)$ with γ.

6. Conclusions

This paper proposed a new robust stabilization method for uncertain state-delayed systems which can be less conservative than existing results. An algorithm involving convex optimization was also proposed to construct a controller with a suboptimal upper bound of the delay such that the system can be stabilized for all admissible uncertainties. It was shown by numerical examples that the proposed delay-dependent stabilization method can even capture the delay-independent stabilizability of the system, which is not possible with the existing results. It was also shown that a stabilizable set of systems can be enlarged using a delayed feedback control for a case where the information on the size of the delay is available.

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