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Explicit Input and Output Feedback Control for Discrete-Time Systems

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Abstract: A new form of output feedback control, referred herein as explicit input and output feedback control (EIOC), is proposed for linear discrete-time systems. Unlike the conventional dynamic output feedback control described by a state-space model, the proposed EIOC has a batch form, where current control is explicitly expressed using current and past system outputs and past control inputs over a recent time horizon. The paper formulates the EIOC law and discusses its features and desirable characteristics. The EIOC is shown to be equivalent to static output feedback control for an augmented system. The coefficients of the EIOC are obtained to achieve the H_∞ performance criterion. Finally, numerical examples are presented to illustrate the effectiveness of the EIOC.

Keywords: Dynamic control, explicit input and output feedback control, output feedback control.

1. INTRODUCTION

Dynamic controllers represented by state-space models have been the main type of output feedback controller used for developing control laws for linear discrete-time systems [1-7]. Controllers of this type have been designed to satisfy various criteria and objectives such as H_2, H_∞, pole assignment, etc. Such controllers, however, lack transparency, structural flexibility, and some desirable properties.

They lack transparency because the state space control law does not show clearly how previous samples of system inputs and outputs contribute to present control since this information is embedded in the state equations. Structural inflexibility results from the fact that once the order of the controller is fixed, the number of free design parameters in the controller is also fixed. Existing fixed-order state space controllers inherently have this structural inflexibility [3,9]. More importantly, state space controllers have the infinite impulse response (IIR) structure. According to the literature on filters, the finite impulse response (FIR) structure is more robust against temporary modelling uncertainties than the IIR structure [5-13]. Therefore, controllers of IIR type may be less robust than those of FIR type as illustrated in [6]. Finally, before dynamic controllers can be applied, a state space model must be set up, which is not a direct method of implementing the control law on a microprocessor or a computer.

Motivated by the need for a controller that would not suffer from the above limitations, we introduce a new type of controller, which explicitly and directly uses the current and past outputs as well as past control inputs. We show how the proposed controller, referred to as explicit input and output feedback control (EIOC), avoids the above limitations.

In Section 2, we introduce the structure of the EIOC and its properties, and show the equivalence of the EIOC to a static output feedback controller for an augmented system. The coefficients of the EIOC are obtained to meet the H_∞ criterion in Section 3. Numerical examples are provided in Section 4 to illustrate the effectiveness of the EIOC. Finally, conclusions of the work are provided in Section 5.

2. EXPLICIT INPUT AND OUTPUT FEEDBACK CONTROL

Consider the discrete-time system described by

$$\begin{aligned} x_{k+1} &= Ax_k + B_w w_k + B_u u_k, \\ z_k &= C_z x_k + D_{zw} w_k + D_{zu} u_k, \\ y_k &= C_y x_k + D_{yw} w_k, \end{aligned} \tag{1}$$

where $x \in \Re^n$, $u \in \Re^{n_u}$, $y \in \Re^{n_y}$, $z \in \Re^{n_z}$, and $w \in \Re^{n_w}$ are the state, the control input, the measured output, the controlled output, and the exogenous disturbance input, respectively.

The proposed EIOC has the following structure:

$$u_k = \sum_{i=k-N_y}^{k} H_{k-i} y_i + \sum_{i=k-N_u}^{k-1} L_{k-i} u_i, \tag{2}$$

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where H_i and L_i are coefficients to be determined later, and N_y and N_u are referred to as the output horizon length and the control horizon length, respectively. It is noted that the total number of parameter to be chosen is $N_y + N_u + 1$.

The proposed EIOC has some features which are not directly present or evident in the conventional dynamic controllers represented by state space models. These features are briefly mentioned below.

(a) The EIOC of form (2) is intuitive and its practical implementation is straightforward. It just needs to store several past values of the measured outputs and control inputs, and compute the control law (2), which is very suitable for direct implementation on a microprocessor or a computer. The EIOC shows directly how the past measured outputs and control inputs contribute to the current control values, unlike the state space controllers.

(b) The proposed EIOC can realize both IIR type controllers and FIR type ones in a unified framework. The latter type is known to be more robust to temporary uncertainties than the former type, and this aspect is illustrated later in Section 4. If neither of the horizon lengths is zero, an IIR type controller is obtained, as given in (2). If N_u is zero, the FIR type controller is obtained as follows:

$$u_k = \sum_{i=k-N_y}^{k} H_{k-i} y_i. \tag{3}$$

If both N_y and N_u are set to zero, the EIOC reduces to a well-known simple static output feedback control, i.e., $u_k = H_0 y_k$.

(c) The EIOC provides better design flexibility than the conventional state space controllers with respect to the tradeoff between controller complexity and controller performance. To elaborate, let us consider SISO systems. A full order controller obtained through a conventional method can be represented, after transformation into an observable or controllable canonical form, as follows:

$$\begin{aligned} \eta_{k+1} &= A_c \eta_k + B_c y_k, \\ u_k &= C_c \eta_k + D_c y_k, \end{aligned} \tag{4}$$

where $\eta_k \in \Re^{N_c}$ is the state vector of the controller. The N_c-th order dynamic controller has N_c independent parameters in A_c, and another N_c parameters in either B_c or C_c, depending on whether the controller is in the observable or controllable canonical form, respectively. Finally there is one parameter in D_c giving a total of $2N_c+1$ controller parameters. Recalling that all state space models can be transformed into control or observer canonical forms, we can easily see how many independent parameters are needed in terms of transfer functions. It follows then that only odd number of parameters is permitted in a controller represented by a state space model. As a result, we cannot freely choose a state space controller with a specific number of parameters to achieve the desired closed-loop performance without undue increase in the order (complexity) of the controller. On the other hand, the EIOC allows flexility in choosing an odd or even number of parameters in the controller. For example, choosing $N_u = 1$ and $N_y = 2$ enables us to obtain an EIOC with four parameters ($N_u+N_y+1=4$), which would have complexity and performance between first and second order state space controllers ($2N_c + 1=3$, $2N_c+1=5$). The EIOC also permits choosing any combination of past outputs or control inputs. This feature will be illustrated through numerical examples later in the paper.

The EIOC in (2) can be written more compactly as

$$u_k = H_0 y_k + HY_k + LU_k, \tag{5}$$

where the gains are given by $H = [H_1 H_2 \cdots H_{N_y}]$ and $L = [L_1 L_2 \cdots L_{N_u}]$, and Y_k and U_k are defined by

$$Y_k \triangleq \left[y_{k-1}^T \, y_{k-2}^T \cdots y_{k-N_y}^T \right]^T,$$

$$U_k \triangleq \left[u_{k-1}^T \, u_{k-2}^T \cdots u_{k-N_u}^T \right]^T.$$

As a first step toward the augmentation of EIOC with the system (1), we represent (5) as a dynamic system with states U_k and Y_k as follows:

$$\begin{aligned} Y_{k+1} &= E_y Y_k + F_y y_k, \\ U_{k+1} &= E_u U_k + F_u u_k \\ &= (E_u + F_u L) U_k + F_u H Y_k + F_u H_0 y_k, \end{aligned} \tag{6}$$

where E_y and F_y are defined as

$$E_y \triangleq \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ I_{N_y} & 0 & 0 & \cdots & 0 \\ 0 & I_{N_y} & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{N_y} & 0 \end{bmatrix}, \quad F_y \triangleq \begin{bmatrix} I_{N_y} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

E_u and F_u are similarly defined by replacing I_{N_y} in E_y and F_y with I_{N_u}, respectively. It is noted that $N_y = 1$ gives $E_y = 0_{N_y \times N_y}$ and $F_y = I_{N_y}$. Similarly $N_u = 1$ gives $E_u = 0_{N_u \times N_u}$ and $F_u = I_{N_u}$.

Substituting the EIOC (5) into the system (1) and augmenting U_k and Y_k with the system state vector using (6), we obtain the following closed-loop system:

$$\begin{aligned} \xi_{k+1} &= A_{cl} \xi_k + B_{cl} w_k, \\ z_k &= C_{cl} \xi_k + D_{cl} w_k, \end{aligned} \tag{7}$$

where $\xi_k = [x_k^T \, Y_k^T \, U_k^T]^T$ is the state vector, and A_{cl}, B_{cl}, C_{cl}, and D_{cl} are defined as

$$A_{cl} \triangleq \begin{bmatrix} A + B_u H_0 C_y & B_u H & B_u L \\ F_y C_y & E_y & 0 \\ F_u H_0 C_y & F_u H & E_u + F_u L \end{bmatrix}, \tag{8}$$

$$B_{cl} \triangleq \begin{bmatrix} B_w + B_u H_0 D_{yw} \\ F_y D_{yw} \\ F_u H_0 D_{yw} \end{bmatrix}, \tag{9}$$

$$C_{cl} \triangleq \left[C_z + D_{zu} H_0 C_y \;\; D_{zu} H \;\; D_{zu} L \right], \tag{10}$$

$$D_{cl} \triangleq D_{zw} + D_{zu} H_0 D_{yw}. \tag{11}$$

Combining control gain matrices into a single matrix $\mathscr{K} = [H_0 \quad H \quad L]$ and introducing the following equivalent open-loop system matrices

$$\mathscr{A} = \begin{bmatrix} A & 0 & 0 \\ F_y C_y & E_y & 0 \\ 0 & 0 & E_u \end{bmatrix}, \quad \mathscr{B}_w = \begin{bmatrix} B_w \\ F_y D_{yw} \\ 0 \end{bmatrix}, \tag{12}$$

$$\mathscr{B}_u = \begin{bmatrix} B_u \\ 0 \\ F_u \end{bmatrix}, \quad \mathscr{C}_y = \begin{bmatrix} C_y & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{13}$$

$$\mathscr{C}_z := \begin{bmatrix} C_z & 0 & 0 \end{bmatrix}, \quad \mathscr{D}_{yw} = \begin{bmatrix} D_{yw} \\ 0 \\ 0 \end{bmatrix}, \tag{14}$$

$$\mathscr{D}_{zw} = D_{zw}, \quad \mathscr{D}_{zu} = D_{zu}, \tag{15}$$

we can write the closed-loop systems matrices $\{A_{cl}, B_{cl}, C_{cl}, D_{cl}\}$ in terms of the equivalent open-loop matrices $\{\mathscr{A}, \mathscr{B}_w, \mathscr{C}_z, \mathscr{D}_{zw}\}$ and the overall controller gain matrix $\mathscr{K}$ as

$$A_{cl} = \mathscr{A} + \mathscr{B}_u \mathscr{K} \mathscr{C}_y, \quad B_{cl} = \mathscr{B}_w + \mathscr{B}_u \mathscr{K} \mathscr{D}_{yw},$$
$$C_{cl} = \mathscr{C}_z + \mathscr{D}_{zu} \mathscr{K} \mathscr{C}_y, \quad D_{cl} = \mathscr{D}_{zw} + \mathscr{D}_{zu} \mathscr{K} \mathscr{D}_{yw}.$$

As a result, in order to find $\mathscr{K}$ of the EIOC for the system (1), we have only to find a static output feedback controller, $u_k = \mathscr{K} \bar{y}_k$, for the following augmented system:

$$\xi_{k+1} = \mathscr{A} \xi_k + \mathscr{B}_w w_k + \mathscr{B}_u u_k,$$
$$z_k = \mathscr{C}_z \xi_k + \mathscr{D}_{zw} w_k + \mathscr{D}_{zu} u_k,$$
$$\bar{y}_k = \mathscr{C}_y \xi_k + \mathscr{D}_{yw} w_k,$$

where $\bar{y}_k = [y_k^T \; Y_k^T \; U_k^T]^T$ can be regarded as the measured output of the above system. This result enables us to use the existing static output feedback control design methods to obtain the EIOC. Furthermore, various performance criteria can be taken into account in designing the EIOC. In the next section, we will consider the H_∞ performance criterion in designing the EIOC.

Remark 1: In order to obtain an output feedback controller (3) of FIR type, we have only to set L to zero. $\mathscr{K}$ and other matrices given in (12), (13), (14), and (15) reduce to :

$$\mathscr{K} = \begin{bmatrix} H_0 & H \end{bmatrix}, \quad \mathscr{A} = \begin{bmatrix} A & 0 \\ E_y C_y & F_y \end{bmatrix}, \quad \mathscr{B}_w = \begin{bmatrix} B_w \\ F_y D_{yw} \end{bmatrix},$$

$$\mathscr{B}_u = \begin{bmatrix} B_u \\ 0 \end{bmatrix}, \quad \mathscr{C}_z = \begin{bmatrix} C_z & 0 \end{bmatrix}, \quad \mathscr{C}_y = \begin{bmatrix} C_y & 0 \\ 0 & I \end{bmatrix},$$

$$\mathscr{D}_{yw} = \begin{bmatrix} D_{yw} \\ 0 \end{bmatrix}, \quad \mathscr{D}_{zw} = D_{zw}, \quad \mathscr{D}_{zu} = D_{zu}.$$

Such output feedback controllers of FIR type are known to be more robust against temporary parameter changes than those of IIR type, as mentioned earlier. The design method of FIR type controllers through the state augmentation approach was discussed in [6]. While the proposed method extracts all controller gain matrices separately into a single matrix $\mathscr{K}$ as shown above, the method of [6] does not. As a result, the proposed method leads to a simpler condition for the solution. This, in turn, leads to better results than [6], which will be illustrated in the second numerical example of Section 4.

Remark 2: The controller (2) can be exactly represented as a structured state space model (4). However, such a model with structured matrices A_c, B_c, C_c, and D_c, makes numerical computation intractable. The controllers of the form (2) with general N_y and N_u cannot be obtained though the conventional methods for A_c, B_c, C_c, and D_c with all independent elements. Only for special cases, $N_y = N_u$, we can employ the existing different methods of obtaining reduced or full dynamic output feedback controls corresponding to $N_y = N_u = 1$, $N_y = N_u = 2, \cdots$. However, output feedback controls of the form (2) for $N_y \neq N_u$ cannot be obtained from existing approaches due to structured matrices.

3. APPLICATION OF EIOC TO H_∞ CRITERION

In this section, we consider the H_∞ performance criterion in the design of the EIOC. In order to apply the H_∞ criterion to the controller design, we consider the transfer function of the closed-loop system, which is $T_{cl}(z) = C_{cl}(zI - A_{cl})^{-1} B_{cl} + D_{cl}$. From the well-known bounded real lemma for linear discrete-time systems [1], it is deduced that given $\gamma > 0$, the closed-loop system controlled by the EIOC satisfies $\|T_{cl}\|_\infty < \gamma$, and A_{cl} is stable if and only if there exist $P = P^T > 0$ and $\mathscr{K}$ satisfying the following matrix inequality:

$$\begin{bmatrix} -P & \Xi & \Psi & 0 \\ \star & -P & 0 & (\mathscr{C}_z + \mathscr{D}_{zu} \mathscr{K} \mathscr{C}_y)^T \\ \star & \star & -\gamma I & (\mathscr{D}_{zw} + \mathscr{D}_{zu} \mathscr{K} \mathscr{D}_{yw})^T \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0, \tag{16}$$

where $\Xi = P(\mathscr{A} + \mathscr{B}_u \mathscr{K} \mathscr{C}_y)$, $\Psi = P(\mathscr{B}_w + \mathscr{B}_u \mathscr{K} \mathscr{D}_{yw})$, I is an identity matrix of appropriate dimension, and the symbol $\star$ denotes a transposed block induced by symmetry. The matrix inequality (16) can be rewritten as

$$G + U^T \mathscr{K} V + V^T \mathscr{K} V < 0, \tag{17}$$

where G, U, and V are defined by

$$G \triangleq \begin{bmatrix} -P & P\mathscr{A} & P\mathscr{B}_w & 0 \\ \star & -P & 0 & \mathscr{C}_z^T \\ \star & \star & -\gamma I & \mathscr{D}_{zw}^T \\ \star & \star & \star & -\gamma I \end{bmatrix}, \tag{18}$$

$$U \triangleq \begin{bmatrix} \mathscr{B}_u^T P \; 0 \; 0 \; \mathscr{D}_{zu}^T \end{bmatrix}, \quad V \triangleq \begin{bmatrix} 0 \; \mathscr{C}_y \; \mathscr{D}_{yw} \; 0 \end{bmatrix}. \tag{19}$$

We can eliminate the dependence on $\mathscr{K}$ of the inequality (35) using the following projection lemma [1]:

Lemma 1: Consider a symmetric matrix $G \in \mathbf{R}^{m\times m}$ and two matrices U and V with the column dimension m. Then there exists a matrix $\mathscr{K}$ of compatible dimensions such that $G + U^T \mathscr{K} V + V^T \mathscr{K}^T U < 0$ if and only if $W_U^T G W_U < 0$ and $W_V^T G W_V < 0$, where W_U and W_V denote any bases of the null space of U and V, respectively.

Finally, the EIOC for the H_∞ criterion can be obtained as in the following theorem:

Theorem 1: Given the discrete-time system (1), there exists an EIOC that satisfies the H_∞ criterion with $\gamma > 0$ if and only if there exist symmetric matrices $P > 0$ and $Q > 0$ that satisfy the following conditions:

$$\Theta^T \left[\begin{array}{cc|c} \mathscr{A}Q\mathscr{A}^T - Q & \mathscr{A}Q\mathscr{C}_z^T & \mathscr{B}_w \\ \star & -\gamma I + \mathscr{C}_z Q \mathscr{C}_z^T & \mathscr{D}_{zw} \\ \hline \star & \star & -\gamma I \end{array}\right] \Theta < 0, \tag{20}$$

$$\Sigma^T \left[\begin{array}{cc|c} \mathscr{A}^T P\mathscr{A} - P & \mathscr{A}^T P\mathscr{B}_w & \mathscr{C}_z^T \\ \star & -\gamma I + \mathscr{B}_w^T P \mathscr{B}_w & \mathscr{D}_{zw}^T \\ \hline \star & \star & -\gamma I \end{array}\right] \Sigma < 0, \tag{21}$$

$$P = Q^{-1}, \tag{22}$$

where Σ and Θ are given by

$$\Sigma = \left[\begin{array}{c|c} N_P & 0 \\ \hline 0 & I \end{array}\right], \quad \Theta = \left[\begin{array}{c|c} N_Q & 0 \\ \hline 0 & I \end{array}\right], \tag{23}$$

and N_Q and N_P denote the bases of the null spaces of $[\mathscr{B}_u^T \; \mathscr{D}_{zu}^T]$ and $[\mathscr{C}_y \; \mathscr{D}_{yw}]$, respectively.

Proof: First, in order to eliminate $\mathscr{K}$, we apply the projection lemma [1]. With U and V defined in (19), W_U and W_V can be represented as:

$$W_U = \left[\begin{array}{cc|cc} P^{-1} & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & I \\ 0 & I & 0 & 0 \end{array}\right] \left[\begin{array}{c|cc} N_Q & 0 & 0 \\ \hline 0 & I & 0 \\ 0 & 0 & I \end{array}\right],$$

$$W_V = \left[\begin{array}{cc|cc} 0 & 0 & I & 0 \\ I & 0 & 0 & 0 \\ \hline 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{array}\right] \left[\begin{array}{c|cc} N_P & 0 & 0 \\ \hline 0 & I & 0 \\ 0 & 0 & I \end{array}\right].$$

If Q is set to P^{-1}, $W_U^T G W_U < 0$ and $W_V^T G W_V < 0$ reduce to (43) and (46), respectively. This completes the proof.

The EIOC is formulated as a static output feedback control problem and it also has a nonlinear coupling constraint, $P = Q^{-1}$, which is hard to satisfy. However, we can find P and Q satisfying (20), (21), and (22) by utilizing the method presented in [2]. Accordingly, the coupling constraint, $P = Q^{-1}$, is replaced by

$$\begin{bmatrix} P & I \\ I & Q \end{bmatrix} \geq 0. \tag{24}$$

We then equate P to Q^{-1} as closely as possible by solving the cone complementary problem, which is represented as:

$$\min \quad \mathrm{tr}(PQ) \text{ subject to } (20), (21), \text{and } (24). \tag{25}$$

The problem in (25) can be solved iteratively using the cone complementary algorithm. For iterative computation, we can construct an algorithm and then obtain a suboptimal performance bound, γ_{so}, as follows:

Algorithm for suboptimal H_∞ bound with EIOC:

Step 1: Choose a sufficiently large initial $\gamma > 0$ such that there exists a feasible solution to (20), (21) and (24). Set $\gamma_{so} = \gamma$.

Step 2: Set i to zero. Find a feasible set (P_0, Q_0) satisfying (20), (21) and (24).

Step 3: Solve the following linear matrix inequality (LMI) problem for the variables P and Q:

$$\begin{array}{ll} \text{Minimize} & \mathrm{tr}(Q_i P + P_i Q) \\ \text{subject to} & (20), (21), \text{and } (24). \end{array} \tag{26}$$

Set $P_{i+1} = P$ and $Q_{i+1} = Q$.

Step 4: If the stopping criterion is satisfied, then set $\gamma_{so} = \gamma$ and return to Step 2 after decreasing γ. If the stopping criterion is not satisfied within a specified number of iterations, say $i_{\max}$, then exit. Otherwise, set $i = i+1$ and go to Step 3.

The dimension of P is $(n + N_y \times n_y + N_u \times n_u) \times (n + N_y \times n_y + N_u \times n_u)$. Therefore, according to [2], the stopping criterion in Step 4 is to check whether the solution to the minimization problem given in (26) satisfies $\mathrm{tr}(PQ) = n + N_y \times n_y + N_u \times n_u$. However, satisfying this equality exactly is numerically too demanding. Instead, we check whether the matrix P, the solution to (26), satisfies the inequality

$$\Theta^T \left[\begin{array}{cc|c} \mathscr{A}P^{-1}\mathscr{A}^T - P^{-1} & \mathscr{A}P^{-1}\mathscr{C}_z^T & \mathscr{B}_w \\ \star & -\gamma I + \mathscr{C}_z P^{-1} \mathscr{C}_z^T & \mathscr{D}_{zw} \\ \hline \star & \star & -\gamma I \end{array}\right] \Theta < 0. \tag{27}$$

If P satisfies the above inequality, then we set $Q = P^{-1}$. The original three conditions (20), (21), and (22) are then all satisfied. After P is obtained, the inequality (16) reduces to an LMI with respect to $\mathscr{K}$, and the remaining variables can be found by solving the LMI.

4. NUMERICAL EXAMPLES

Two numerical examples are presented in this section to illustrate the effectiveness of the proposed design methods. Especially, the features (b) and (c) in Section 2 are highlighted through the examples.

Table 1. Suboptimal performance bounds γ_{so} for different horizon lengths (Example 1).

	Proposed method	Method of [6]
$N_y = 0,\ N_u = 0$	11.39	11.40
$N_y = 1,\ N_u = 0$	11.22	11.22
$N_y = 2,\ N_u = 0$	11.06	11.06
$N_y = 3,\ N_u = 0$	10.91	10.91
$N_y = 4,\ N_u = 0$	10.78	10.79
$N_y = 1,\ N_u = 1$	9.95	×
$N_y = 2,\ N_u = 1$	9.90	×
$N_y = 3,\ N_u = 1$	9.88	×
$N_y = 1,\ N_u = 2$	9.90	×
$N_y = 1,\ N_u = 3$	9.88	×
$N_y = 2,\ N_u = 2$	9.87	×
$N_y = 3,\ N_u = 3$	9.87	×

Example 1: Consider the following system:

$$x_{k+1} = \begin{bmatrix} 1 & 1 & 0.2 \\ -0.1 & 1.1+\delta_k & 0 \\ 0 & 0.2 & 0.5+\delta_k \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix} w_k + \begin{bmatrix} 0 \\ 0.1 \\ 0 \end{bmatrix} u_k, \tag{28}$$

$$z_k = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_k,$$

$$y_k = [0 \;\; 1 \;\; 0] x_k + 2w_k,$$

where δ_k is an uncertain parameter that is employed to evaluate robustness of the controller. Initially, we assumed $\delta_k = 0$.

The optimal performance bound, i.e., γ^*, for this system when controlled by a full order dynamic controller was computed to be 9.87 using the results of [1,12], whose algorithm is provided through a Matlab command "dhinflmi". We applied the proposed algorithm to find the EIOC for different horizon lengths. The corresponding suboptimal performance bounds, γ_{so}, are shown in Table 1. $i_{\max}$ is set to be 2000. When the proposed algorithm did not yield a feasible solution even after $i_{\max}$ iterations, we assumed that the problem was infeasible for the given γ_{so}. When the EIOC is dependent only on the measured outputs and not on the past control inputs, its performance improved as the output horizon length, N_y, increased. In this case, the resultant controllers were of the FIR type. When N_y = 4, the performance bound improved up to γ =10.78, which was still far from the optimal value γ^* = 9.87. Since the increase in the horizon length implies computational burden during execution, a large N_y should be avoided. We also obtained the suboptimal performance bounds using the method presented in [6]. Those values are listed in Table 1. It is noted that the method of [6] can only be applied to the design of controllers of the FIR type. As such, it cannot be applied to the design of controllers of the IIR type. For SISO systems, the proposed method seemed to be comparable to [6]. However, as will be shown in the next example, the proposed method outperformed the method of [6] for MIMO systems. It was also observed from Table 1 that the performance improved significantly when we used a single past control input u_{k-1} even though the output horizon length was set to be as short as 1. This was because the controller switched from the FIR type to the IIR type when we used the past control u_{k-1}. For $N_y = N_u = 2$, the resultant EIOC achieved the optimal value, $\gamma^* =$ 9.87. As a compromise between controller complexity and controller performance, it was reasonable to choose $N_y = 2$ and $N_u = 1$, which yielded $\gamma_{so} = 9.90$. Note that $N_u = N_y = 1$ gave an EIOC corresponding to a first order state space controller with three parameters ($N_u + N_y + 1 = 2N_c + 1 = 3$), while $N_u = N_y = 2$ corresponded to a second order state space controller with five parameters ($N_u + N_y + 1 = 2N_c + 1 = 5$). Output feedback controls for $N_u = N_y = 1$ and $N_u = N_y = 2$ can also be obtained from existing reduced order output feedback controls. Note that the EIOC for $N_y = 2$ and $N_u = 1$ was a controller with a good compromise between complexity and performance which is not available by any state space controller. The EIOC for $N_y = 2$ and $N_u = 1$ is:

$$u_k = -3.8879 y_k + 3.9566 y_{k-1} + 0.0582 y_{k-2} + 0.9760 u_{k-1}. \tag{29}$$

At the sacrifice of optimality, we can achieve robustness. When $N_y = 4$ and $N_u = 0$, we obtained the EIOC of FIR type as follows:

$$u_k = -4.1152 y_k + 0.0514 y_{k-1} + 0.0872 y_{k-2} + 0.0392 y_{k-3} + 0.1675 y_{k-4}. \tag{30}$$

The performance bound for this case was γ_{so} = 10.78, which was worse than any IIR type EIOC. However, as already mentioned, the FIR structure is known to be more robust than the IIR structure against transient uncertainties. To illustrate the robustness of the EIOC, we assumed the following transient uncertainty in (28).

$$\delta_k = \begin{cases} 0.4, & 100 \le k \le 150, \\ 0, & \text{otherwise.} \end{cases}$$

The initial state of the system is given by $x_0 = [0\ 1\ 0]^T$. We assumed that $w_k \in \mathcal{N}(0, 0.01^2)$ and performed simulations for 300 steps using the EIOC of the IIR type in (29) and the EIOC of the FIR type in (30). Fig. 1 compares the trajectories of the second state, where the solid line presents for the EIOC of the FIR type (29) and the dotted one presents the EIOC of the IIR type (30). Both trajectories deviate from steady state values when uncertainties are applied. However, it is clearly observed that the trajectory of the EIOC of the FIR type converges to zero faster with much less deviation than that of the EIOC of the IIR type.

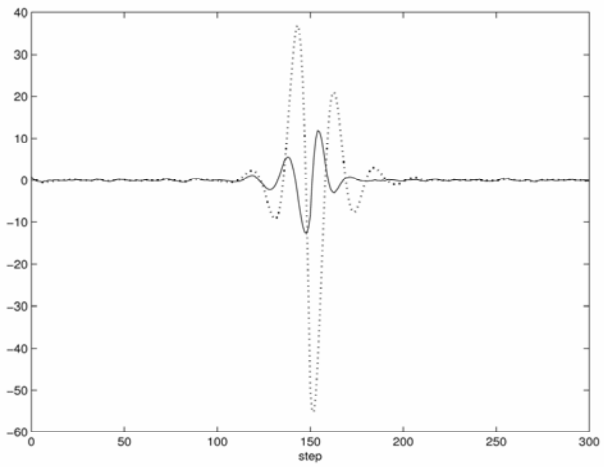

Fig. 1. Trajectory comparison of the second state (solid: EIOC of the FIR type, dotted: EIOC of the IIR type).

Table 2. Suboptimal performance bounds γ_{so} for different horizon lengths (Example 2).

	Proposed method	Method in [6]
$N_y = 0, N_u = 0$	7.20	7.21
$N_y = 1, N_u = 0$	5.34	5.34
$N_y = 2, N_u = 0$	4.90	4.99
$N_y = 3, N_u = 0$	4.79	4.94
$N_y = 4, N_u = 0$	4.75	4.93
$N_y = 1, N_u = 1$	4.67	×

Example 2: Consider the following multi-input unstable system

$$x_{k+1} = \begin{bmatrix} 2.9 & 0.3 & 2 \\ 1 & 0 & 1 \\ 0.3 & 0.6 & -0.6 \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} w_k + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u_k,$$

$$z_k = \begin{bmatrix} x_k \\ u_k \end{bmatrix}, \quad y_k = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_k.$$

Table 2 shows the suboptimal performance bounds for different horizon lengths. The optimal performance bound γ^* for this system was computed to be 4.67. As in the previous example, the performances of the EIOC with $N_u = 0$ improved as the output horizon length, N_y, increased. For MIMO systems, the proposed method outperforms the method of [6] especially when the horizon length increases. It is notable that the performance bound of the EIOC with $N_y = N_u = 1$ is equal to that of the optimal dynamic output feedback control, which is of second order. It turns out that a further increase in the horizon lengths does not result in the performance improvement. The EIOC for $N_u = 1$ and $N_y = 1$ is given by

$$u_k = \begin{bmatrix} 1.2331 \\ 0.5017 \end{bmatrix} y_k + \begin{bmatrix} 0.4362 \\ 0.4156 \end{bmatrix} y_{k-1} + \begin{bmatrix} 0.4999 & 0.5065 \\ 0.4739 & 0.6506 \end{bmatrix} u_{k-1}. \tag{31}$$

5. CONCLUSION

A new controller structure for discrete-time systems, referred to as explicit input and output feedback controller (EIOC), was proposed in this paper. This structure has some features not present in the standard state space controllers, including transparency, ease of implementation, ability to represent both FIR and IIR type controllers, and flexible trade off between complexity and performance by specification of the number of independent controller parameters without any constraints.

The EIOC was shown to be equivalent to a static output feedback control for an augmented system. This result allowed the use of available design techniques based on static output feedback controls to the EIOC. In an example of application, the H_∞ criterion was employed to design an EIOC, and the simulation results were presented. The results of this paper can also be used to design an EIOC that can meet other types of performance criteria.

It is not the intention of this paper to claim that the EIOC can entirely replace the standard state space type controllers, as there are many situations where the latter would be more useful than the former. Instead, we are suggesting that the EIOC has features that are not available in state space type controllers, and that these features may be useful in certain applications.

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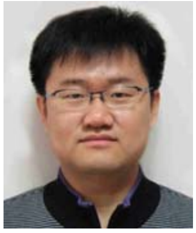

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