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A minimum variance FIR filter with an H_∞ error bound and its application to the current measuring circuitry

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ABSTRACT

In this paper, we propose a minimum variance finite impulse response (FIR) filter with a guaranteed H_∞ error bound. The proposed FIR filter is required in advance to make use of only inputs and outputs on the recent finite time so that its impulse response has a finite duration, or it has finite memory with respect to past data. From such requirement, a transfer function from the external noises to the estimation error is first obtained, and then the corresponding estimation error variance is minimized with respect to the filter gains while keeping its H_∞ norm bounded. The constrained minimization problem is represented with linear matrix inequalities (LMI) that can be efficiently solved using convex programming techniques. It is shown through application to the current measuring circuitry in the magnetic levitation system that the proposed FIR filter is more robust against temporary uncertainties than an existing mixed H_2/H_∞ IIR filter with a guaranteed H_∞ error bound.

1. Introduction

Estimation problems have been widely dealt with in sciences and engineering areas. Specially, state estimation has been considered as an important research issue for dynamic systems since full information on the state is not available in most cases. For state estimation, two types of filters, *i.e.*, finite impulse response (FIR) and infinite impulse response (IIR) filters, have been researched for a long time and their properties become generally known.

Recently, FIR filters for state estimation have received a lot of attention due to its good inherent properties arising in the FIR structure as its heavy computation burden is alleviated by the fast computer technologies. Furthermore, some trials have been made to extend the FIR filters even to more complicated nonlinear and hybrid systems. It is practically acknowledged and shown via simulations that the FIR filters are more robust than IIR filters when applied to systems with temporary modeling uncertainties and round-off errors in computation. Since the FIR filters put much weight on the recent data and discard old data, they are likely to avoid divergence and poor tracking. Additionally, FIR filters are designed from finite dimensional optimization, and hence multi criteria and any constraints can be easily applied. Until now, such nice FIR filters have employed a minimum variance criterion that is very tractable for mathematical analysis and has useful physical meanings by providing the dimension of energy and putting much weights on large errors [1–9].

As another useful and commonly-used criterion, the H_∞ criterion is often used for the worst case design, which is roughly defined by a gain between the total energies of the disturbances and the estimation errors [10–12]. In order to optimize the performance on the average and diminish the effect of the worst case simultaneously, the H_∞ criterion has often been employed together with a minimum variance criterion in designing IIR filters [13–16]. The authors proposed FIR filters for both the H_∞ and minimum variance criteria under the strong assumption that a system matrix is nonsingular [17]. However, to the best of the authors' knowledge, there is no result on the minimum variance FIR filters with a guaranteed H_∞ bound without the nonsingularity condition of the system matrix.

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In this paper, an FIR filter for estimation of the state x_k at time k is obtained to be linear with inputs $u_{k+\cdot}$ and outputs $y_{k+\cdot}$ on the recent finite time $[k-N, k-1]$ so that its impulse response has a finite duration and hence it can be represented as

$$\hat{x}_k = \sum_{i=k-N}^{k-1} H_{k-i} y_i + \sum_{i=k-N}^{k-1} L_{k-i} u_i, \tag{1}$$

for some gains H_{k-i} and L_{k-i}. Note that N is the order of the filter, and H_{k-i} and L_{k-i} will be determined according to the criterion. From a linear filter with the finite impulse response as in (1), a transfer function from external noises to the estimation error is first obtained, and then the corresponding estimation error variance is minimized with respect to filter gains, H_{k-i} and L_{k-i}, under the restriction that the H_∞ error bound is less than a prescribed value. This optimization problem is converted to a constrained minimization one represented with linear matrix inequalities (LMI) that can be efficiently solved using convex programming techniques. Unlikely the existing result [17], the nonsingularity condition of the system matrix is not required, which leads to more stable numerical computation and more applications.

To demonstrate the validity, we apply the proposed FIR filter to the current measuring circuitry in the magnetic levitation system. It is shown that the proposed FIR filter is more robust against temporary uncertainties than an existing H_2/H_∞ IIR filter with a guaranteed H_∞ error bound.

This paper is organized as follows. In Section 2, a minimum variance FIR filter with an H_∞ error bound is represented with LMIs. In Section 3, a real application to the current measuring circuitry is given. Finally, the conclusions are presented in Section 4.

2. A minimum variance FIR filter with an H_∞ error bound

Consider a linear discrete-time state space model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, \\ y_k &= Cx_k + Dw_k, \end{aligned} \tag{2}$$

where $x_k \in \Re^n$ is the state, $u_k \in \Re^l$ is the input, $y_k \in \Re^q$ is the output, and $w_k \in \Re^p$ is a vector disturbance containing both process noises and measurement noises, respectively. The pair (A, C) is observable. w_k is assumed to have unit variance. When the H_∞ error bound is computed, w_k is considered as a deterministic disturbance, which belongs to l_2 space. The following system with independent noise sources

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, \\ y_k &= Cx_k + D\upsilon_k, \end{aligned} \tag{3}$$

can be transformed as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + [G \quad 0]\begin{bmatrix} w_k \\ \upsilon_k \end{bmatrix}, \\ y_k &= Cx_k + [0 \quad D]\begin{bmatrix} w_k \\ \upsilon_k \end{bmatrix}. \end{aligned} \tag{4}$$

which leads to the model (2). We can say that the model (2) is not limited.

The FIR filter (1) can be written as follows:

$$\hat{x}_k = HY_{k-1} + LU_{k-1}, \tag{5}$$

where H, L, Y_{k-1}, and U_{k-1} are given by

$$\begin{aligned} H &\triangleq [H_N \quad H_{N-1} \quad \cdots \quad H_1], \\ L &\triangleq [L_N \quad L_{N-1} \quad \cdots \quad L_1], \\ U_{k-1} &\triangleq [u_{k-N}^T \quad u_{k-N+1}^T \quad \cdots \quad u_{k-1}^T]^T, \\ Y_{k-1} &\triangleq [y_{k-N}^T \quad y_{k-N+1}^T \quad \cdots \quad y_{k-1}^T]^T, \end{aligned}$$

and the order of the filter, N is assumed to be greater than or equal to the system order n. Why we need this assumption, $N \geqslant n$, will be explained later on. By using the following definitions:

$$\widetilde{C}_N \triangleq \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix},$$

$$\widetilde{B}_N \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ CB & 0 & \cdots & 0 & 0 \\ CAB & CB & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{N-2}B & CA^{N-3}B & \cdots & CB & 0 \end{bmatrix},$$

$$\widetilde{G}_N \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ CG & 0 & \cdots & 0 & 0 \\ CAG & CG & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{N-2}G & CA^{N-3}G & \cdots & CG & 0 \end{bmatrix},$$

$$\widetilde{D}_N \triangleq \text{diag}\underbrace{\{D, D, \cdots, D\}}_{N},$$

$$\begin{aligned} L_b &\triangleq [A^{N-1}B \quad A^{N-2}B \quad \cdots \quad B], \\ L_g &\triangleq [A^{N-1}G \quad A^{N-2}G \quad \cdots \quad G], \\ W_{k-1} &\triangleq [w_{k-N}^T \quad w_{k-N+1}^T \quad \cdots \quad w_{k-1}^T]^T, \end{aligned}$$

the recent measurements, *i.e.*, Y_{k-1}, over $[k-N, k]$ and the current state, *i.e.*, x_k, can be represented by

$$x_k = A^N x_{k-N} + L_b U_{k-1} + L_g W_{k-1}, \tag{6}$$

$$Y_{k-1} = \widetilde{C}_N x_{k-N} + \widetilde{B}_N U_{k-1} + (\widetilde{G}_N + \widetilde{D}_N) W_{k-1}. \tag{7}$$

Replacing Y_{k-1} in (5) with (7) and arranging terms yield

$$\hat{x}_k = H\widetilde{C}_N x_{k-N} + (H\widetilde{B}_N + L)U_{k-1} + H(\widetilde{G}_N + \widetilde{D}_N)W_{k-1}. \tag{8}$$

The H_∞ error bound of a filter is given as

$$\sup_{w_i} \frac{\sum_{i=N}^{\infty} (x_i - \hat{x}_i)^T (x_i - \hat{x}_i)}{\sum_{i=N}^{\infty} w_i^T w_i}, \tag{9}$$

for nonzero disturbance belonging to l_2 space. Furthermore, the estimation error, $\hat{x}_i - x_i$, of a H_∞ filter should approach zero asymptotically for zero disturbance, *i.e.*, $w_i = 0$. Since we are concerned with a guaranteed finite H_∞ error bound, we make x_k in (6) equal to $\hat{x}_k$ in (8) for zero disturbance, $W_{k-1} = 0$. Otherwise, the H_∞ error bound goes to infinity. A guaranteed finite H_∞ error bound requires the following conditions:

$$H\widetilde{C}_N = A^N, \quad L = L_b - H\widetilde{B}_N. \tag{10}$$

Finally, we rewrite the FIR filter in (5) as

$$\hat{x}_k = HY_{k-1} + (L_b - H\widetilde{B}_N)U_{k-1}. \tag{11}$$

It is noted that once H is obtained, L is automatically computed from (10).

From (6), (7), and (11), the estimation error, $e_k = \hat{x}_k - x_k$, can be represented as follows:

$$\begin{aligned} e_k &= \hat{x}_k - x_k, \\ &= HY_{k-1} + (L_b - H\widetilde{B}_N)U_{k-1} - A^N x_{k-N} - L_b U_{k-1} \\ &\quad - L_g W_{k-1}, \\ &= [H(\widetilde{G}_N + \widetilde{D}_N) - L_g]W_{k-1}, \end{aligned} \tag{12}$$

where the last equality comes from the conditions (10). W_{k-1} in (12) can be written as

$$W_k = A_u W_{k-1} + B_u w_k, \tag{13}$$

where A_u and B_u are given by

$$A_u = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I \end{bmatrix}. \tag{14}$$

Putting (12) and (13) together, we have a transfer function $T(z)$ from the disturbances w_k to the estimation errors e_k,

$$T(z) = [H(\widetilde{G}_N + \widetilde{D}_N) - L_g](zI - A_u)^{-1}B_u. \tag{15}$$

From now on, we will obtain H in (15) so that $T(z)$ is optimized according to the minimum variance criterion with a prescribed H_∞ error bound. First, the estimation error variance is represented in terms of H and the corresponding LMI is obtained for making it possible to make tractable numerical computation with another constraints.

The estimation error variance is given by

$$\mathbf{E}[e_k^T e_k] = \frac{1}{2\pi}\mathrm{tr}\left(\int_{-\pi}^{\pi} T^*(e^{jw})T(e^{jw})dw\right) = \frac{1}{2\pi}\|T(z)\|_2. \tag{16}$$

It is well known that $\|T(z)\|_2$ is given by

$$\|T(z)\|_2 = \sqrt{\mathrm{tr}\left([H(\widetilde{G}_N + \widetilde{D}_N) - L_g]P[H(\widetilde{G}_N + \widetilde{D}_N) - L_g]^T\right)}. \tag{17}$$

where P is the controllability Grammian computed from

$$P = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i,$$

or from $A_u P A_u^T - P + B_u B_u^T = 0$ [18]. We can easily see through simple algebraic calculation that $P = I$.

By introducing a matrix variable W such that

$$W > [H(\widetilde{G}_N + \widetilde{D}_N) - L_g][H(\widetilde{G}_N + \widetilde{D}_N) - L_g]^T, \tag{18}$$

we have only to minimize $\mathrm{tr}(W)$ for optimizing the minimum variance criterion since $\mathrm{tr}(W) > \|T(z)\|_2^2$. By the Schur complement, the inequality (18) is equivalent to

$$\begin{bmatrix} W & H(\widetilde{G}_N + \widetilde{D}_N) - L_g \\ (\widetilde{G}_N + \widetilde{D}_N)^T H^T - L_g^T & I \end{bmatrix} > 0. \tag{19}$$

Since a filter gain matrix H in (19) satisfies $H\widetilde{C}_N = A^N$, it is parameterized by

$$H = FM + A^N(\widetilde{C}_N^T\widetilde{C}_N)^{-1}\widetilde{C}_N^T, \tag{20}$$

where M^T is composed of the bases of the null space of $\widetilde{C}_N^T$, and F is a new matrix variable containing the independent variables. It is noted that $\widetilde{C}_N^T\widetilde{C}_N$ is guaranteed to be nonsingular since (A, C) is observable and N is assumed to be greater than or equal to n.

Finally, the minimum variance criterion for minimizing $\mathbf{E}[e_k^T e_k]$ can be represented by

$$\begin{aligned} &\min_{F,W} \mathrm{tr}(W) \text{subject to} \\ &\begin{bmatrix} W & \Xi_N^T \\ \star & I \end{bmatrix} > 0, \end{aligned} \tag{21}$$

where Ξ_N is defined by

$$\Xi_N \triangleq (FM + A^N(\widetilde{C}_N^T\widetilde{C}_N)^{-1}\widetilde{C}_N^T)(\widetilde{G}_N + \widetilde{D}_N) - L_g. \tag{22}$$

Through long and tedious calculation, we can show that the optimal H and cost for the minimum variance criterion can be represented in a closed-form from a constrained optimization problem, which is stated in the following theorem.

Theorem 1. *The optimal H for the minimum variance criterion is given in a closed-form as*

$$\begin{aligned} H = &A^N(\widetilde{C}_N^T\Lambda_N^{-1}\widetilde{C}_N)^{-1}\widetilde{C}_N^T\Lambda_N^{-1} + L_g(\widetilde{G}_N + \widetilde{D}_N)^T\Lambda_N^{-1}[I \\ &- \widetilde{C}_N(\widetilde{C}_N^T\Lambda_N^{-1}\widetilde{C}_N)^{-1}\widetilde{C}_N^T\Lambda_N^{-1}] \end{aligned}$$

where Λ_N is defined by

$$\Lambda_N \triangleq (\widetilde{G}_N + \widetilde{D}_N)(\widetilde{G}_N + \widetilde{D}_N)^T,$$

and the corresponding minimum error variance is

$$\mathrm{tr}(H\Lambda_N H^T - 2L_g(\widetilde{G}_N + \widetilde{D}_N)^T H^T + L_g L_g^T).$$

It is noted that the result of Theorem 1 is more general than that of [19] since the correlation between system and measurement noises are allowed in this paper.

If we are interested in the minimum variance criterion only, we can use the result in Theorem 1. However, an LMI representation in (21) is more convenient for imposing another constraint such as an H_∞ error bound. We have only to add another LMI for an H_∞ error bound. Next, we will impose the H_∞ error bound using the well-known *bounded real lemma*.

For the system transfer function $G(z) = \bar{C}(zI - \bar{A})^{-1}\bar{B}$, it is well known from the *bounded real lemma* that, given $\gamma > 0$, the following two conditions are equivalent:

(1) $\|G(z)\|_\infty < \gamma$.
(2) There exists an $X > 0$ such that

$$\begin{bmatrix} -X & X\bar{A} & X\bar{B} & 0 \\ \bar{A}^T X & -X & 0 & \bar{C}^T \\ \bar{B}^T X & 0 & -\gamma I & 0 \\ 0 & \bar{C} & 0 & -\gamma I \end{bmatrix} < 0.$$

From the following correspondences,

$$\bar{A} \leftarrow A_u,$$
$$\bar{B} \leftarrow B_u,$$
$$\bar{C} \leftarrow (FM + A^N(\widetilde{C}_N^T \widetilde{C}_N)^{-1} \widetilde{C}_N^T)(\widetilde{G}_N + \widetilde{D}_N) - L_g,$$

we obtain the following LMI for imposing the H_∞ error bound:

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ \star & -X & 0 & \Xi_N \\ \star & \star & -\gamma_\infty I & 0 \\ \star & \star & \star & -\gamma_\infty I \end{bmatrix} < 0, \quad (23)$$

where Ξ_N is defined in (22).

Putting all pieces together, we can formulate the minimum variance FIR filter with a guaranteed H_∞-norm bound. The final result is summarized in the following theorem.

Theorem 2. *Given $\gamma_\infty > 0$, assume that the following LMI problem is feasible:*

$$\min_{W,S,X>0,F} \ \mathrm{tr}(W) \ \text{subject to}$$
$$\begin{bmatrix} W & \Xi_N^T \\ \star & I \end{bmatrix} > 0,$$
$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ \star & -X & 0 & \Xi_N \\ \star & \star & -\gamma_\infty I & 0 \\ \star & \star & \star & -\gamma_\infty I \end{bmatrix} < 0.$$

Then the gain matrix of the minimum variance FIR filter with guaranteed H_∞-norm bound, γ_∞, is given by

$$H = FM + A^N(\widetilde{C}_N^T \widetilde{C}_N)^{-1} \widetilde{C}_N^T,$$

where M^T is composed of the bases of the null space of $\widetilde{C}_N^T$.

The minimum variance FIR filter obtained from Theorem 2 allows us to design the optimal FIR filters with respect to the error variance while guaranteeing a prescribed performance level in the H_∞ sense. For the feasibility of LMIs, γ_∞ should satisfy $\gamma_\infty \geqslant \gamma_\infty^*$, where γ_∞^* is the optimal H_∞-norm of the FIR filter of the form (1). The value γ_∞^* can be obtained from the LMI (23).

It is noted that as the order of the filter, N, increases, the performance becomes better. However, the computation load increases with the order of the filter. There is an inevitable trade-off between the performance and the computation load. It would be desirable to choose the minimum N insofar as the required performance can be achieved in terms of H_2 and H_∞ criteria.

In the next section, we give a numerical example to illustrate the usefulness of the proposed FIR filter.

3. Numerical example

To illustrate the performance of the proposed FIR filter, a magnetic levitation system in Fig. 1 is taken as a numerical example. The main purpose of this system is to levitate an object in the air using an electromagnet. By changing the voltage input appropriately and hence inducing the current flowing in the coil, the magnetic force is generated from the electromagnet such that the object can float in the air.

Driving circuits of electromagnets are usually implemented using an H-bridge circuit in Fig. 2. To protect components inside, drivers usually have over-current protection circuits. One of commonly used methods is to use a shunt resistor to measure the magnitude of current. Since the driver circuit is subject to transistor switching noises, we need a filter to estimate the magnitude of the current. Here, it is shown that the proposed FIR filter is useful for estimating the current.

To begin with, let us represent the magnetic levitation system with the state space model (2). The current passing through the inductor and the resistor associated with the electromagnet is obtained from the following differential equation:

$$i(R + R_s) + L\frac{di}{dt} = V, \quad (24)$$

where i is the current, V is the voltage input, L is the inductance of the electromagnet, R is the resistance of the electromagnet, and R_s is the resistance of the shunt resistor. Discretizing the differential Eq. (24) yields

$$i_{k+1} = \left(1 - \frac{R + R_s}{L}\Delta\right) i_k + \frac{\Delta}{L} V_k,$$

where Δ is the sampling time. Since the inductance of the electromagnet is affected by the characteristics of the levitated object, we reflect it in the form of uncertainties and noises, and hence obtain the following model:

$$i_{k+1} = \left(1 - \frac{R + R_s}{L}\Delta + \delta_k\right) i_k + \frac{\Delta}{L} V_k + 0.01 w_{1,k},$$

where δ_k and $w_{1,k} \sim \mathcal{N}(0, 1)$ are the parameter uncertainties and the process noises, respectively. The current

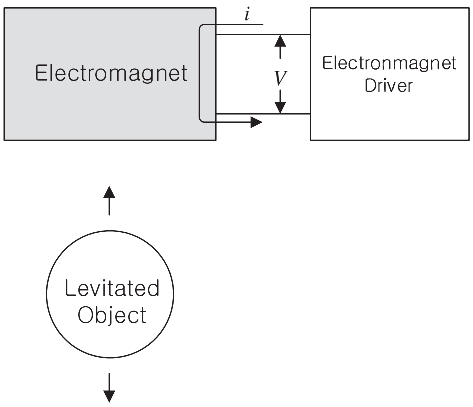

Fig. 1. A magnetic levitation system.

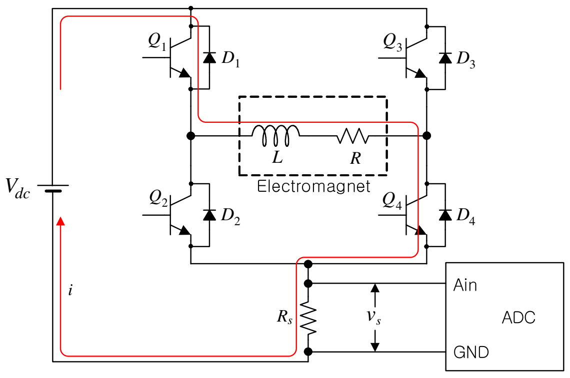

Fig. 2. An electromagnet driver with current measuring circuitry (ADC: analog-to-digital converter).

flowing through the electromagnet is measured indirectly by measuring the voltage drop across the shunt resistor. From Kirchhoff's law, it follows that we have $v_s = R_s i$, where v_s is the voltage drop across the shunt resistor. Since this voltage drop is measured by an analog-to-digital converter (ADC), it is reasonable to consider conversion latency. We reflect this latency as one-step delay, which leads to $v_{s,k+1} = R_s i_k$. If the conversion latency of an ADC is greater than one step size, we can increase two or more step delay. For example, the measurement equation $y_{s,k+2} = R_s i_k$ can be used instead of $y_{s,k+1} = R_s i_k$.

Finally, the overall model is represented in the form of (2) as follows:

$$\begin{bmatrix} v_{s,k+1} \\ i_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & R_s \\ 0 & 1 - \frac{R+R_s}{L}\Delta + \delta_k \end{bmatrix} \begin{bmatrix} v_{s,k} \\ i_k \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\Delta}{L} \end{bmatrix} V(k) + \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \end{bmatrix} \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix},$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_{s,k} \\ i_k \end{bmatrix} + \begin{bmatrix} 0 & 0.01 \end{bmatrix} \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix},$$

where y_k is the voltage measurement obtained through the ADC and $w_{2k} \sim \mathcal{N}(0,1)$ is the external noise. Physical values of electric components and the sampling time are chosen as follows: $L = 0.1$ [H], $R = 4$ [Ω], $R_s = 0.05$ [Ω], $\Delta = 0.002$.

We designed an FIR filter using Theorem 2 with the horizon size and an H_∞ error bound set to 8 and 1.05 γ_∞^*, respectively, where $\gamma_\infty^* = 0.1169$. The resultant FIR filter turned out to have $\|T(z)\|_2 = 0.00651$ and $\|T(z)\|_\infty = 0.12272$, while the conventional mixed H_2/H_∞ IIR filter has $\|T(z)\|_2 = 0.02449$ and $\|T(z)\|_\infty = 0.10627$. The IIR filter is better than the proposed FIR filter with respect to norm values. However, FIR filters have their own features such as the robustness against temporary parameter variation, compared to IIR filters. To illustrate the robust performance for unexpected changes or uncertainties, we assume that the system is subject to temporary parameter variation represented by

$$\delta_k = \begin{cases} 0.1, & 100 \leqslant k \leqslant 140 \\ 0, & \text{otherwise} \end{cases}.$$

We apply the constant voltage input, $u = 5$. Current estimation errors due to the proposed FIR filters in Theorem 2 and the conventional mixed H_2/H_∞ IIR filters [13] are compared in Fig. 3. We can see that the estimation error of the proposed FIR filter is smaller than that of the IIR filter on the interval where temporary parameter variation exists. Moreover, it is shown that the estimation error of the proposed FIR filter converges more rapidly than that of the IIR filter when temporary parameter variation disappears. So, the proposed FIR filter can be said to be more robust than IIR filters when applied to systems with temporary modeling errors. Such robust performance improvement comes from the finite memory structure, or the FIR structure. It is also observed from the simulation results that FIR filters have some degraded performance in normal and transient situations. Practically, robustness comes before degraded nominal performance.

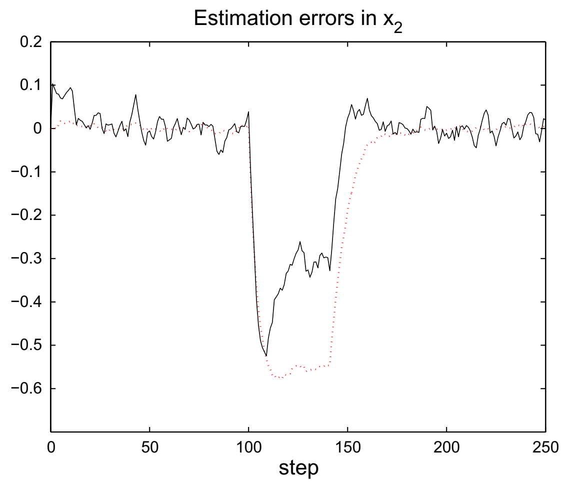

Fig. 3. Current estimation errors: comparison between IIR filters (dotted) and FIR filters (solid).

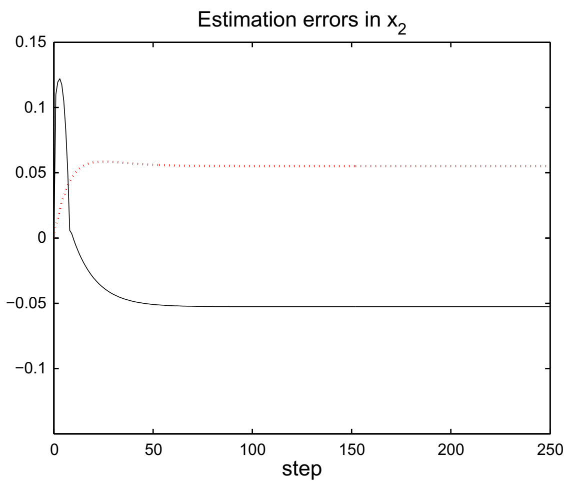

Fig. 4. Current estimation errors from constant disturbances applied to the system: comparison between IIR filters (dotted) and FIR filters (solid).

To illustrate the performance against deterministic disturbances, we apply constant disturbances $w_{1,k} = w_{2,k} = 1$. Actually, this performance is highly related to the H_∞ error bound. Fig. 4 compares the current estimation errors of a conventional mixed H_2/H_∞ IIR filter and the proposed FIR filter. The figure shows that the proposed FIR filter has the estimation error as small as that of the IIR filter, which implies that the proposed FIR filter is as insensitive to constant disturbance as the IIR filter. The transient response of the proposed FIR filter looks a little poor, which comes from the fact that FIR filters work N times after the initial time 0. If an existing IIR filter is adopted in the beginning when FIR filters do not work well, we can avoid such a temporary performance degradation.

To be summarized, we can say that the proposed FIR filter achieves the more robustness against model uncertainties without any sacrifice of the capability with respect to disturbance rejection.

4. Conclusions

In this paper, a minimum variance FIR filter with a guaranteed H_∞ error bound is proposed without any artificial condition and it is applied to the current measuring circuity. Since the optimization problem related to the design of the proposed FIR filter is represented in terms of linear matrix inequalities (LMI), it can be efficiently solved using convex programming techniques. We suggested a circuit system as a good application of the proposed filters to show that FIR structure gives a good robust performance against model uncertainties and a prescribed H_∞ error bound can reduce the effect of the deterministic disturbance on the performance. This application would be a good research topic for improving the current measuring circuitry.

Since the proposed FIR filter has practical features in consideration of both the worst case performance and the average one through the H_∞ error bound and the minimum variance criterion, we believe that many applications in circuits and systems are waiting to be applied. As a future work, it would be meaningful to extend the result of this paper to time varying systems and find more applications.

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